A geometric description of the Grover's algorithm

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1 Introduction

This paper concerns the Grover algorithm that permits to make amplification of quantum states previously tagged by an Oracle. Grover's algorithm allows searches in an unstructured database of n entries, finding a marked element with a quadratic speedup. The algorithm requires a predefined number of runs to succeed with probability close to one.

We try to provides a description of the amplitude amplification quantum algorithm mechanism in a very short computational way based on tensor products providing a geometric presentation of the successive system states. All the basis changes are fully described to provide an alternative to the wide spread Grover description based only on matrices and complex tensor computation. Our experiments encompass numerical evaluations of circuit using the Qiskit library of IBM that meet the theoretical considerations.

Let us consider an unsorted finite set B spanning a Hilbert space $E = Span(B)$ and a function

$$\begin{aligned} f: \quad & B \rightarrow \{0;1\} \\ & x \rightarrow f(x) \end{aligned}$$

This functions characterizes the subsets of B with

$$E_i = span(x \in B / f(x) = i)$$

And $E = E_0 \oplus E_1$

The problem consist in finding at least one $x \in E_1$ avoiding the costly enumeration of all element of B one by one (if no extra information is available on B) is the corner stone in computer science of advanced data structures. Grover's algorithm provides a quadratic speed-up and received a considerable of attention due to this characteristic. Not that the algorithm introduced by Grover in 1996 permits a simultaneous evaluation of all the SAT solutions to find the correct assignment with a promise of a quadratic speedup.

2 Basis switches

The Hadamard gates are operations that maps the basis state $B(|0\rangle;|1\rangle)$ into $B(|p\rangle;|m\rangle)$ by a $\pi/2$ rotation on $Y - axe$ and a $2.\pi$ rotation on $X - axe$ and creates an equal superposition of the two basis states: $H.|0\rangle = \frac{1}{\sqrt{2}}.(|0\rangle + |1\rangle) = |+\rangle$, $H.|1\rangle = \frac{1}{\sqrt{2}}.(|0\rangle - |1\rangle) = |-\rangle$, $H.|+\rangle = |0\rangle$, $H.|-\rangle = |1\rangle$.

The X-gate is a symmetry around the $\pi/4$ axis leading to the following basic transformations: $X.|0\rangle = |1\rangle$, $X.|1\rangle = |0\rangle$, $H.|+\rangle = |+\rangle$, $H.|-\rangle = -|-\rangle$

The Grover algorithm is composed of 4 steps:

- The first one is the initialization that defines $|\psi_1\rangle = |p\rangle^{\otimes n} \otimes |m\rangle$;
- The second one consists in the definition of the Oracle that mark the solution by changing the amplitude of the marked state defining now a quantum state spanned by $|p\rangle^{\otimes n}$ and $|r\rangle$: $|\psi_4\rangle = |p\rangle^{\otimes n} - \frac{2}{\sqrt{2^n}}.|r\rangle$ (figure 1)
- The third step consists in switching to $B(|1\rangle^{\otimes 5}; |r\rangle)$ that is the only basis available to make the apply the $CC..CZ - gate$ responsible of the amplification.
- The last one is no more that the measurement.

For a geometric point of view, we have drawn a parallelogram in the plane $B\left(|p\rangle^{\otimes n}; |r\rangle\right)$ as stressed on figure 1.

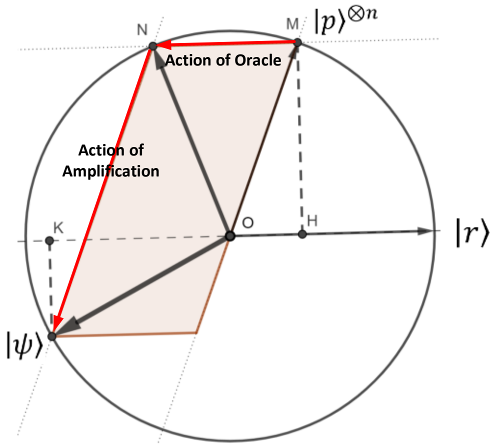

FIG. 1**.** Geometric representation of the Grover's algorithm

3 Concluding remarks

In this paper we investigate a description of the Grover's algorithm using geometric considerations and a tensorial computations to gives a more readable algorithmic description close to the classical algorithmic description commonly used in the OR community.

4 References

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