MILP formulations for continuous set-covering on networks

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1 Introduction

Covering problems are well-studied in the domain of Location Science. When the location space is a network, the most frequent assumption is to consider the candidate facility locations, the points to be covered, or both, to be finite sets. In this work, we study the set-covering location problem when both candidate locations and demand points are continuous on a network. This variant generalizes several problems, including set-covering, and uncapacitated facility location problems. However, this problem has received little attention, and the scarce existing approaches have focused on particular cases, such as tree networks and integer covering radius.

Here we study the general problem through an integer programming approach. To the best of our knowledge, the only existing Mixed Integer Linear Programming formulation is due to [1], which assumes that after a preprocessing algorithm, the lengths of the edges of the network are less than the covering radius. This MILP formulation indexes candidate locations by edges, and uses a covering condition between pairs of edges (one is a demand, and the other indexes location).

We present new MILP formulations [2]. We introduce the concept of residual covers on vertices and give a new covering condition based on the residual covers contributed by candidate locations. Moreover, we index candidate locations by edges and vertices, so we can use a preprocessing algorithm to reduce the search space. Based on these results, we propose the first MILP formulation and several presolving algorithms to reduce the size of the MILP, especially in tightening big-M constants, and reducing redundant constraints. Moreover, a second MILP is proposed, which allows edge lengths greater than the covering radius. As opposed to the existing formulation of the problem (including the first MILP proposed herein), the number of variables and constraints of this second model does not depend on the lengths of the network's edges. This second model represents a scalable approach that particularly suits real-world networks, whose edge lengths are usually greater than the covering radius.

Finally, we implement the existing and new MILP formulations in JuMP modeling language. Our computational experiments show the strengths and limitations of our exact approach to solving both real-world and random networks. Our formulations are also tested against an existing exact method.

2 Summary and comparison of MILP models

We compare the sizes of our first MILP formulation (without any preprocessing) and the MILP formulation from [1] on a network $G = (V, E)$.

Based on the new covering condition, we have four MILP models `F0`, `F`, `SF` and `RF`, which essentially have the same structure of variables and constraints. Based on the new covering condition, we can use the preprocessing algorithm called 'Delimitation' technique to reduce the number of constraints in the MILP model. In addition, we have 'Strengthening' techniques

	Variables		Constraints
	Binaries	Continuous	
Our MILP formulation	$\|V\|^2 + 2(\|V\|\|E\| + \|V\| + \|E\|)$	$\|V\| + \|E\|$	$\|E\|^2 + \|V\|^2 + 5\|E\|\|V\| + 7\|E\| + 3\|V\|$
MILP formulation in [1]	$\|E\|^3 + 3\|V\|\|E\| + \|E\|$	$3\|V\|\|E\| + \|E\|$	$3\|E\|^3 + 8\|E\|\|V\| + \|E\|$

TAB. 1 – Comparative summary on MILP formulations

Model	Delimitation	Strengthening	Long edge	Size	Input network	Comment
`EF`	No	No	No	Very large	Subdivided network	From [1]
`F0`	No	No	No	Large	Subdivided network	The simple model
`F`	Yes	No	No	Medium	Subdivided network	The complete model
`SF`	Yes	Yes	No	Medium	Subdivided network	The strengthened model
`RF`	Yes	Yes	Yes	Small	Degree-two-free network	The reduced model

TAB. 2 – Model summary

to tighten the big-M bound of the MILP model. Finally, we use the ‘Long edge’ technique to model the covering problem on networks with edge lengths greater than the covering radius.

The computational results and source code are publicly released on our project website : https://github.com/lidingxu/cflg/.

3 Conclusions

We will study valid inequalities and constraint propagation for the continuous set-covering problem. Since the MILP models contain auxiliary continuous/integer variables, the future study can apply the Benders-like algorithm to project out these variables. Open problems involve : the convertibility of these MILP models to other location problems, e.g. obnoxious facility location ; the possibility to restrict the continuous search space to its subspace or tractable discrete point set.

Références

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