A bilvevel pricing and routing problem

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1 Introduction

The Profitable Tour Problem (PTP) belongs to the class of Vehicle Routing Problems with profits. In PTP, a vehicle, starting from a central depot, can visit a subset of the available customers, collecting a specific revenue whenever a customer is visited. The objective of the problem is the maximization of the net profit, i.e., the total collected revenue minus the total route cost. Most of the literature in this field considers only one decision maker. However, in several real-world routing applications, and in particular in the last-mile delivery, there are different involved agents with conflicting goals. If the decisions are made in a hierarchical order, this problem can be modeled with bilevel programming, with the PTP at the lower level.

In this paper, we consider a company, which acts as a "leader" and offers disjoint subsets of a given set of items to a set of independent drivers. At the lower level, each driver solves a PTP communicating to the company the items she accepts to serve. Both the company and the drivers aim at maximizing their net profit, which is calculated differently in the two levels. We further propose two bilevel formulations that model this interaction allowing the leader not only to anticipate the best followers' response, but also to find the optimal pricing scheme for each carrier. The value function reformulations of the bilevel models are considered and further reformulated by projecting out some of the lower-level variables. We find exact solutions to these models using a branch and cut approach, leveraging on an alternative reformulation of the lower-level problems.

2 Problem description

We consider a single-leader multiple-follower Stackelberg game in which there is a set of items I that need to be delivered to a corresponding set of customers V. An intermediary company, acting as a leader, receives a prize p_i for each item to be delivered. Given a set K of potential carriers (e.g., occasional drivers), the intermediary searches for carriers that can deliver these packages, and pays to carrier $k \in K$ a price $\bar{p}_i^k$ for each delivered item, i.e., for each served customer ($0 \leq \bar{p}_i^k < p_i$). The leader has to pack the items (the customers) into $|K|$ disjoint subsets, say $(P_1, \ldots, P_{|K|})$, and make a proposal for each carrier with respect to the chosen items (customers) in P_k. The leader makes also a proposal about the compensation paid for each item. Specifically, the price $\bar{p}_i^k$ offered to carrier k for item i is chosen by setting the margin the leader wants to keep over the received price p_i. We assume that there are $|M|$ different values of the margin the leader can choose from (expressed as percetange to be applied to the price p_i). The higher is the margin, the higher is the compensation received by the leader. However, the higher is the margin, the lower is the attractiveness of the item for the follower. We also assume that the leader cannot assign more than a certain amount of items to each carrier.

In turn, each carrier $k \in K$ receives the proposal, and, based on her net profit, decides on a subset of customers $Q_k \subseteq P_k$ to accept to serve. Hence, each follower is solving a Profitable Tour Problem with respect to the given set of items (customers) P_k. Each carrier can refuse to deliver some items, in which case the intermediary's revenue for this item becomes zero. The goal of the leader is making a call to the carriers, so as to maximize his revenue, which is defined as

$$\sum_{k \in K} \sum_{i \in Q_k} (p_i - \bar{p}_i^k).$$

2.1 Mathematical formulation

We define $\bar{c}_{ij}^k$ the travel time for carrier k to go from i to j, b^k as the upper bound on the number of items the leader can assign to carrier k and p_{mi} the price gained by the leader when applying margin m to item i. The problem can be formulated by using the following decision variables :

— x_i^k : 1, iff the intermediary offers to the carrier k to serve customer i
— y_i^k : 1, iff carrier k accepts to serve customer i
— α_{mi}^k, 1 iff the selected margin for item i to be offered to carrier k is m
— z_{ij}^k : 1 iff carrier k goes from i to j.

The bilevel problem can be formulated as :

$$\max_{x,\alpha} \quad \sum_{k \in K} \sum_{i \in V} \sum_{m \in M} p_{mi} \alpha_{mi}^k y_i^k \tag{1}$$

$$\sum_{k \in K} x_i^k \leq 1 \forall\, i \in V \setminus \{0\} \tag{2}$$

$$\sum_{i \in V} x_i^k \leq b^k \forall\, k \in K \tag{3}$$

$$\sum_{m \in M} \alpha_{mi}^k = x_i^k \forall\, i \in V, k \in K \tag{4}$$

$$y^k \in \arg \Phi^k(x^k, \alpha^k) \forall\, k \in K \tag{5}$$

$$x^k, \alpha_m^k \in \{0,1\}^{n+1} \forall\, m \in M, k \in K \tag{6}$$

where $\Phi^k(x^k, \alpha^k)$ is the optimal solution value of the k-th follower problem, which, for a given $(\tilde{x}^k, \tilde{\alpha}^k)$ is formulated as :

$$\Phi^k(\tilde{x}^k, \tilde{\alpha}^k) = \max_{y,z} \sum_{i \in V} \left[p_i - \sum_{m \in M} (p_{mi} \tilde{\alpha}_{mi}^k) \right] y_i^k - \sum_{(i,j) \in A} \bar{c}_{ij}^k z_{ij}^k \tag{7}$$

$$y_i^k \leq \tilde{x}_i^k \forall\, i \in V \tag{8}$$

$$(y^k, z^k) \text{ is a route} \tag{9}$$

$$y^k \in \{0,1\}^{n+1}, z^k \in \{0,1\}^{|A|} \tag{10}$$

where A is the set of all possible links between i and j. We provide a single level reformulation of the former bilevel model as well as two linearization techniques for the non-linear objective function (1). We also provide a reformulation which projects out z variables. Tests are made on instances derived from benchmark instances for the Orienteering Problem (OP). We compare the performance of the formulations and test their effectiveness according to different problem parameters.