Optimization methods for the multi-commodity flow blocker problem

Isma Bentoumi[1,3], Fabio Furini[2], A. Ridha Mahjoub[1], Sébastien Martin[3]

[1] LAMSADE, UMR CNRS 7243, University Paris Dauphine PSL, Paris, France
`{ridha.mahjoub, isma.bentoumi}@lamsade.dauphine.fr`

[2] Department of Computer Control and Management Engineering Antonio Ruberti, Roma, Italy
`fabio.furini@uniroma1.it`

[3] Huawei Technologies France, Boulogne-Billancourt, France.
`{sebastien.martin, isma.bentoumi}@huawei.com`

Mots-clés : *Combinatorial optimization, blocker problem, interdiction problem, multi-commodity flow problem, branch-and-cut, polyhedral analysis.*

1 Introduction

We are interested in evaluating the strength of a network by determining the maximum number of failures that it can face. This can be done by solving a multi-commodity flow blocker problem.
Given a directed graph $G = (V, A)$, where V is the set of vertices and A the set of arcs, let K be a set of commodities in G. With every arc $a \in A$ is associated a capacity $c_a \in \mathbb{Z}_+$ and a flow cost $p_a \in \mathbb{Z}_+$. Every commodity $k \in K$ is defined as a triplet (s_k, t_k, d_k) where $s_k \in V$ is the source, $t_k \in V$ is the destination and $d_k \in \mathbb{Z}_+$ the bandwidth, i.e., the amount of flow that should pass from s_k to t_k. Considering such network, the multi-commodity flow problem (MCFP) consists in finding a set of arcs, called multicommodity flow, routing flow to satisfy all demands, with respect to flow conservation constraints and capacity constraints. The MCFP aims to minimize the total flow cost. We study the blocker problem applied to the multi-commodity flow problem, which is called the multi-commodity flow blocker problem (MCFBP). Let $r_a \in \mathbb{Z}_+$ be an interdiction cost associated with every arc $a \in A$, the MCFBP consists in finding a set of arcs, with a minimum total interdiction cost, to remove from the graph in such a way that the minimum cost of the multi-commodity flow in the remaining graph is greater than or equal to a given threshold. The threshold is called *target cost value* and it is denoted by Φ.

We illustrate the features of optimal MCFBP solutions thanks to the graph shown in Figure 1 composed by 8 vertices, 12 arcs and 2 commodities $k_1 = (s_1, t_1, d_1 = 8)$, $k_2 = (s_2, t_2, d_2 = 10)$. We report on each arc two values separated by the symbol " ;" : the first one, in blue, is the capacity of the arc ; the second one, in black, is the cost for sending one unit of flow through the arc. The optimal MCFP solution in this network consists in sending 8 units of flow from s_1 to t_1 through path $\{(s_1, v_1), (v_1, v_3), (v_3, t_1)\}$ and 10 units of flow from s_2 to t_2 through path $\{(s_2, v_2), (v_2, v_4), (v_4, t_2)\}$ with a total cost equal to 268. We assume that all arcs have the same value of interdiction costs, equal to 1, and consider a target cost value equal to 300. An optimal solution of the MCFBP consists in removing the arc (s_2, v_2), represented with a dashed line. The minimum cost of the multi-commodity flow remaining in the graph is equal to 418, which is greater than 300.

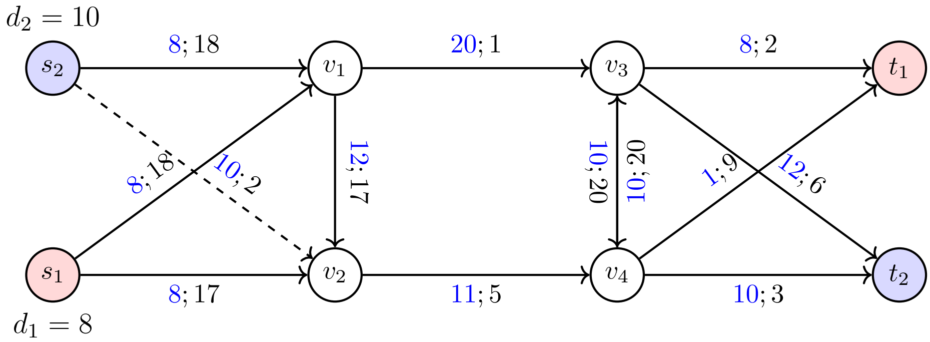

FIG. 1 – Example of an optimal solution for the MCFBP with a target cost value $\Phi = 300$.

2 Solution approach

We introduce a new IP model for the multi-commodity flow blocker problem. Our goal is to provide a thin and well-understood formulation. This formulation has an exponential number of constraints called cover constraints. Let z_a be a binary variable associated with the set of arcs A of a graph G, each variable encoding whether the corresponding arc is removed from G or not. Model (1) solves a multi-commodity flow blocker problem.

$$\min \quad \sum_{a \in A} r_a w_a \tag{1a}$$

$$\sum_{a \in mcf} w_a \geq 1 \qquad mcf \in MCF, \tag{1b}$$

$$w_a \in \{0, 1\} \qquad a \in A, \tag{1c}$$

where MCF is the set of all multi-commodity flows in G with a total cost less than the target cost value.

We examine the polyhedral structure of formulation (1) and give new valid inequalities to strengthen the model. Using this we develop a Branch-and-cut algorithm to solve model (1).

Références

[1] Magnouche, Y. and S. Martin (2020). Most vital vertices for the shortest s−t path problem : complexity and branch−and−cut algorithm. Optimization Letters 14, 2039–2053.

[2] Lim, C. and J. C. Smith (2007). Algorithms for discrete and continuous multicommodity flow network interdiction problems. IIE Transactions 39 (1), 15–26.