Operations Research Approaches for the Satellite Constellation Design Problem : Exact and Heuristic Methods

Luca Mencarelli[1], Julien Floquet[2], Frédéric Georges[2]

[1] UMA, ENSTA Paris, Institut Polytechnique de Paris, 91120 Palaiseau, France
luca.mencarelli@ensta-paris.fr
[2] DTIS, ONERA, Université Paris Saclay, 91123 Palaiseau, France
{julien.floquet,frederic.georges}@onera.fr

Mots-clés : *satellite constellation design problem, discontinuous coverage, mixed integer (non)linear programming, feasibility pump.*

1 The satellite constellation design problem

The satellite constellation design problem with discontinuous coverage is an emerging research area arising in aerospace applications (see, for instances, [1]). In this problem we aim to design a constellation comprised of the minimum number of satellites guaranteeing observability of several targets on the Earth surface with a constrained revisit time.

We introduce a set of binary variables $\xi_{i,t,m}$ indicating if the satellite i observes target $m \in \mathcal{X}$ at time t (the time is discretized with a time-step dt over the duration T) and the variables σ_i representing the set of orbital parameters of satellite i. Let

- Δt define the length of a revisit time interval, so that the allowed duration between two consecutive observations of the same target is at most $2\Delta t$,
- $[s] := \{1, 2, \ldots, s\}$ be the set of satellites, $[T]$ the set of time steps, and $[\lfloor T/\Delta t \rfloor]$ the set indexing the revisit time intervals,
- lat_m and $long_m$ be the latitude and the longitude of the target $m \in \mathcal{X}$, respectively,
- $lat(\sigma_i, t)$ and $long(\sigma_i, t)$ be the latitude and longitude of the satellite $i \in [s]$ at time t,
- Σ_i be the feasible set of the orbital parameters of satellite $i \in [s]$,
- θ_i^{max} be the maximum observation angle for satellite $i \in [s]$, and
- $\mathcal{T}(k, \Delta t) := \{t_k, t_{k+1}, t_{k+2}, \ldots, t_{k+\Delta t/dt-1}\}$.

We aim to find the minimum integer s such that :

$$\begin{aligned}
&\sum_{\substack{t \in \mathcal{T}(k,\Delta t) \\ i \in [s]}} \xi_{i,t,m} \geq 1 && \forall k \in [\lfloor T/\Delta t \rfloor], \forall m \in \mathcal{X} \\
&\xi_{i,t,m} = 0 \Longrightarrow \max\{|lat_m - lat(\sigma_i, t)|, |long_m - long(\sigma_i, t)| \cos(lat_m)\} \geq \theta_i^{max} && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X} \\
&\xi_{i,t,m} \in \{0, 1\} && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X} \\
&\sigma_i \in \Sigma_i && \forall i \in [s] \,.
\end{aligned}$$

2 Exact algorithm

We suitably discretize the set of the feasible orbital parameters of a satellite i, by considering $|\mathcal{K}_i|$ possible values ($\mathcal{K}_i \subseteq \Sigma_i$). We solve the problem for iteratively increasing values of s :

$$\begin{aligned}
&\sum_{\substack{t \in \mathcal{T}(k,\Delta t) \\ i \in [s]}} \xi_{i,t,m} \geq 1 && \forall k \in [\lfloor T/\Delta t \rfloor], \forall m \in \mathcal{X} \\
&\xi_{i,t,m} = 0 \Longrightarrow \max\{|lat_m - lat_{i,t,k}|, |long_m - long_{i,t,k}| \cos(lat_m)\} \geq \theta_i^{max} && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X}, \forall k \in \mathcal{K}_i \\
&\xi_{i,t,m} \in \{0, 1\} && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X} \,.
\end{aligned}$$

with $lat_{i,t,k}$ and $long_{i,t,k}$ the discretized values of the satellites latitude and longitude. The second constraint represents the (non)-observability of the m-th target by the i-th satellite at time t. With a proper discretization, the previous problem is equivalent to the original one.

3 Heuristic algorithm

We implemented a Feasibility Pump (mat)heuristic for the satellite constellation design problem. The Feasibility Pump is a well-known algorithm used to provide good feasible solution for mixed integer nonlinear problems (MINLPs). We alternatively solve two problems, namely :

$$
\begin{aligned}
&\min \sum_{t\in[T], i\in[s], m\in\mathcal{X}} \left|\xi_{i,t,m} - \widehat{\xi}_{i,t,m}\right| \\
&\sum_{\substack{t\in\mathcal{T}(k,\Delta t)\\ i\in[s]}} \xi_{i,t,m} \geq 1 && \forall k \in [\lfloor T/\Delta t \rfloor], \forall m \in \mathcal{X} \\
&\xi_{i,t,m} = 0 \Longrightarrow \max\left\{|lat_m - lat_{i,t}|, |long_m - long_{i,t}| \cos(lat_m)\right\} \geq \theta_i^{max} && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X} \\
&\xi_{i,t,m} \in \{0,1\} && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X},
\end{aligned}
$$

and

$$
\begin{aligned}
&\min \sqrt{\sum_{\substack{t\in[T], i\in[s],\\ m\in\mathcal{X}}} \left(\xi_{i,t,m} - \widetilde{\xi}_{i,t,m}\right)^2} + \sqrt{\sum_{t\in[T], i\in[s]} \left(lat(\sigma_i,t) - \widetilde{lat}_{i,t}\right)^2} + \sqrt{\sum_{t\in[T], i\in[s]} \left(long(\sigma_i,t) - \widetilde{long}_{i,t}\right)^2} \\
&\sum_{\substack{t\in\mathcal{T}(k,\Delta t)\\ i\in[s]}} \xi_{i,t,m} \geq 1 && \forall k \in [\lfloor T/\Delta t \rfloor], \forall m \in \mathcal{X} \\
&\max\left\{|lat_m - lat(\sigma_i,t)|, |long_m - long(\sigma_i,t)| \cos(lat_m)\right\} \leq \theta_i^{max} + M\,(1 - \xi_{i,t,m}) && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X} \\
&\xi_{i,t,m} \in [0,1] && \forall i \in [s], \forall t \in [T], \forall m \in \mathcal{X} \\
&\sigma_i \in \Sigma_i && \forall i \in [s]
\end{aligned}
$$

where $\widehat{\xi}_{i,t,m}$ is the solution of the second sub-problem and $(\widetilde{\xi}_{i,t,m}, \widetilde{lat}_{i,t}, \widetilde{long}_{i,t})$ is the solution of the first sub-problem. M is a sufficiently large constant. We solve the couple of previous problems starting from $s = 1$ and progressively increasing s if an iteration limit is reached or the current problem is infeasible. We solve the MILP sub-problem using `IBM ILOG CPLEX 20.1`, and the NLP sub-problem with a MultiStart approach using the `Ipopt` solver. When a feasible solution to the second problem is reached, we verify if the indicator constraints are satisfied : if so, we return the current solution. Then, we apply a MILP-based post-processing to improve the quality of the solution in terms of number of satellites of the constellation.

4 Computational results

Preliminary results with randomly generated targets on the Earth surface show the capabilities of the previous algorithms in terms of CPU time and memory usage. In particular, the exact algorithm suffers from limited scalability when considering a relatively large number of targets. On the other hand, the Feasibility Pump algorithm coupled with the post-processing shows a better scalability potential, but it may not return an optimal solution. Moreover, the post-processing phase is a key algorithmic part to obtain solutions of good quality.

Références

[1] L. Mencarelli, J. Floquet, F. Georges, and D. Grenier. Mixed integer (non)linear approaches for the satellite constellation design problem. *Optimization and Engineering*, 2022.