General edge assembly crossover driven memetic search for split delivery vehicle routing

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1 Introduction

The split delivery vehicle routing problem (SDVRP) [2, 1] is a variant of the conventional vehicle routing problem (VRP). Unlike the VRP where each customer is visited exactly by one vehicle, the SDVRP allows a customer's demand to be split and served by several homogeneous capacitated vehicles starting and finishing at the depot.

Formally, let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected graph where $\mathcal{V} = \{0, 1, \cdots, n\}$ is the vertex set with 0 being the depot and $\mathcal{N} = \{1, \cdots, n\}$ representing n customers and $\mathcal{E}$ is the edge set. Each customer $i \in \mathcal{N}$ is associated with an integer demand $d_i \in \mathcal{Z}^+$. Let $\mathcal{C} = (c_{ij})$ be a non-negative cost (distance) matrix associated with $\mathcal{E}$ satisfying the triangle inequality ($c_{ij} + c_{jk} > c_{ik}$ for all $i, j, k \in \mathcal{V}$ and $i \neq j \neq k$). Given a set of K identical vehicles with capacity Q available at the depot, the SDVRP is to find K routes (K can be limited or unlimited) such that 1) each route starts at the depot to serve a number of customers and ends at the depot without exceeding the vehicle capacity Q, 2) the demand d_i of customer $i \in \mathcal{N}$ can be split and served by more than one vehicle, and 3) the total traveling distance of the K routes is minimized. According to the number K of the available vehicles (fleet size), the problem is called the SDVRP-LF (for limited fleet size) if K is fixed or the SDVRP-UF (for unlimited fleet size) otherwise. For the SDVRP-LF, K is fixed to $K_{min} = \lceil (\sum_{i=1}^{n} d_i / Q) \rceil$ to ensure the feasibility of the solution.

2 SplitMA for SDVRP

We propose the SplitMA algorithm, a population-based hybrid genetic algorithm, to solve the SDVRP [3] which utilizes the strengths of both genetic algorithm and local optimization to solve the routing problem efficiently. SplitMA uses mutation to diversify offspring solutions by randomly removing and re-inserting cities, and an advanced pool updating strategy to manage the population. The algorithm was ranked second at the 12th DIMACS Implementation Challenge on Vehicle Routing - SDVRP Track in 2022.

SplitMA starts from an initial population $\mathcal{P}$ constructed by the population initialization procedure. Then the algorithm evolves the population through a number of generations by applying the gEAX crossover, the local optimization procedure and the population updating procedure. Of particular interest is the general edge assembly crossover operator (gEAX) that creates at each generation β offspring solutions by assembling the edges of two parent solutions. The basic concept behind gEAX is to maintain the edges that are shared by the parent solutions and construct new edges based on this shared structure. The rationale behind this approach is that high-quality solutions of these problems typically have a high number of common edges and these common edges form a stable backbone that is highly likely to be a part of the optimal solution. After restoring the feasibility of each offspring solution in terms of customer demand and vehicle capacity, the solution is diversified by the mutation operator and then submitted to local optimization for quality improvement. Finally, each improved solution is

TAB. 1 – Summary of comparative results between SplitMA and reference algorithms.

Pair algorithms	#Instances	*Best*				*Avg.*			
		#Wins	#Ties	#Losses	*p-value*	#Wins	#Ties	#Losses	*p-value*
SDVRP-LF	162	-	-	-	-	-	-	-	-
SplitMA vs. BKS	162	70	75	17	4.28E-09	-	-	-	
SplitMA vs. SplitILS	162	76	74	12	1.11E-12	97	29	36	7.42E-09
SplitMA vs. iVNDiv	99	92	7	0	3.15E-17	-	-	-	-
SplitMA vs. RGTS	88	78	9	1	2.15E-14	79	8	1	2.76E-14
SplitMA vs. SS	49	44	5	0	1.74E-09	-	-	-	-
SplitMA vs. HGA	21	12	8	1	3.09E-03	-	-	-	-
SDVRP-UF	162	-	-	-	-	-	-	-	-
SplitMA vs BKS	162	73	81	8	2.08E-12	-	-	-	-
SplitMA vs. SplitILS	162	82	76	4	4.35E-16	112	33	17	6.24E-18
SplitMA vs. TSVBA	120	105	13	2	8.69E-20	-	-	-	-
SplitMA vs. FBTS	67	67	0	0	1.12E-12	-	-	-	-
SplitMA vs. MAPM	74	62	12	0	1.72E-12	-	-	-	-
SplitMA vs. ABHC	36	34	2	0	1.83E-07	-	-	-	-

used to update the population by the pool updating strategy. For the SDVRP-LF where the fleet size is set to K_{min}, the number of the used vehicles is reduced to this fleet size by emptying some routes if needed. The algorithm stops and returns the best solution when a predefined stopping condition is met (e.g., a maximum cutoff time or maximum number of generations).

3 Computational results

SplitMA is assessed based on four sets of 324 instances. A comparative study is conducted using 9 well-known reference algorithms that have been previously shown to effectively solve the SDVRP. We adopt 9 references for the comparative study. The algorithm terminates when it reaches a maximum of 40,000 iterations, and the best solution found during the run is returned as the final solution. The SplitMA was run 20 times independently to solve each instance with distinct random seeds. Table 1 shows the results of the SplitMA algorithm compared to reference algorithms for the SDVRP-LF in terms of best objective values. SplitMA found 70 new upper bounds for 43% of the instances, matched BKS values for 46%. This is remarkable performance as BKS values are considered the best results from all existing algorithms. Compared to the best heuristic SplitILS, SplitMA obtained 76 and 97 better results for the best and average values respectively.

As to the SDVRP-UF, Table 1 (lower part) shows that our algorithm updates 73 BKS values (new upper bounds) and matches 81 other BKS values. Compared to the best reference algorithm SplitILS [5], our algorithm reports 82 better, 76 equal results, respectively. For the average results, SplitMA obtains 112 better results compared to SplitILS. SplitMA performs much better than the other reference algorithms (weaker than SplitILS) by obtaining the best results for the vast majority of the instances.

Références

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