Avoiding starvation of Wi-Fi access points : the study of 1-extendable sets in graphs

Thomas Begin[1], Pierre Bergé[2], Anthony Busson[1], Carl Feghali[1], Rémi Watrigant[1]

[1] CNRS, ENS de Lyon, Université Claude Bernard Lyon 1, LIP UMR5668, Lyon, France
{thomas.begin,anthony.busson,carl.feghali,remi.watrigant}@ens-lyon.fr
[2] CNRS, Université Clermont Auvergne, LIMOS, Clermont-Ferrand, France
pierre.berge@uca.fr

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1 Introduction

In this article, we present both structural and algorithmic questions on graphs, which are motivated by an application on Wi-Fi networks. We propose not only theoretical answers (exact/approximation algorithms or hardness reductions) but also heuristics which allow us to output quickly a feasible solution for large instances.

We study the performance of CSMA/ CA networks. CSMA/CA is the mechanism used by the nodes to access the radio channel in many wireless network technologies. It aims to prevent collisions, which happens when several nodes transmit at the same time thereby producing harmful interference that may cause transmissions losses.

Graphs stand as a natural model for CSMA/CA wireless networks : the vertices represent the Wi-Fi access points of the network. Two vertices are adjacent if the two corresponding access points are able to detect the transmissions from each others (for example when they are close and there is no barrier between them).

Two vertices on different channels can transmit simultaneously. However, two vertices of the same channel can transmit simultaneously if they are not adjacent in the graph. Thus, a set of instantaneous transmitters of some given channel is an *independent set* of the graph, *i.e.* a set of vertices that are pairwise non-adjacent. We denote by $\alpha(G)$ the size of the maximum independent set (MIS) of a graph G.

This graph model is used to evaluate the network performance. The performance parameter that is often computed is the *throughput* that offers to each vertex, *i.e.* the number of bits per second that a vertex is able to send. The throughput of a vertex v is strongly correlated to the proportion of time p_v this vertex is transmitting. We denote by p_v this quantity for the vertex $v \in V(G)$. It was shown [3] that, under saturation condition, p_v is given in graph G by :

$$p_v = \frac{\sum_{S \in \mathcal{S}(G): v \in S} \theta^{|S|}}{\sum_{S \in \mathcal{S}(G)} \theta^{|S|}}, \tag{1}$$

where θ is the ratio between transmission and listen phase durations and $\mathcal{S}(G)$ is the collection of independent sets of G. When θ tends to infinity, p_v tends to the number of MISs of G containing v ($\#_v\alpha(G)$) divided by the total number $\#\alpha(G)$ of MISs of G : $\lim_{\theta \to +\infty} p_v = \frac{\#_v\alpha(G)}{\#\alpha(G)}$.

In practice, the value of θ tends to be large to ensure an efficient channel use. A node/vertex that does not belong to any MIS will experience starvation, *i.e.* a very low throughput.

Given an undirected graph G, we say that an induced subgraph $G[X]$, $X \subseteq V(G)$, is 1-*extendable* [1] if all vertices of X belong to at least one MIS of $G[X]$. CSMA/CA networks must be designed in such a way that the induced subgraph of each channel is 1-extendable.

2 Results

Recognition of 1-extendability. The first part of our contribution concern the recognition of 1-extendable graphs. In Wi-fi networks, this problem models the verification that a given channel ensures the fairness of each vertex and avoids starvation.

Problem : 1-EXTENDABILITY **Input :** Graph G **Question :** Does every vertex of G belong to an MIS of G ?

We prove that 1-EXTENDABILITY is NP-hard, even on *unit disk graphs*, whose vertices represent disks (Wi-Fi access points) with the same radius (signal range) on a plane and edges connect intersecting disks. This is a natural model for Wi-Fi networks.

Theorem 1 *1-EXTENDABILITY :*
- *is NP-hard, even on subcubic planar graphs and unit disk graphs,*
- *cannot be solved in time* $2^{o(n)}$, $n = |V|$, *on general graphs, under ETH*[1].

Observe that, on the positive side, if one can solve MAXIMUM INDEPENDENT SET in polynomial time on some hereditary family of graphs, then one can find the answer for 1-EXTENDABILITY on this family with n calls to the poly-time algorithm determining the MIS. Hence, 1-EXTENDABILITY can be solved in polynomial time on perfect graphs and chordal graphs for example.

1-Extendable coloring. The second part of our contribution deal with a more general problem which consist in partitioning a graph into 1-extendable sets. This problem models the attribution of channels on a Wi-Fi network to avoid starvations.

Problem : 1-EXTENDABLE COLORING **Input :** Graph $G = (V, E)$, integer k **Question :** Can we find a k-partition $P_1, \ldots, P_k$ of V such that each $G[P_i]$ is 1-extendable ?

1-EXTENDABLE COLORING is naturally NP-hard as 1-EXTENDABILITY corresponds to the case $k = 1$. We show that a partition with $\lfloor 2\sqrt{n} \rfloor$ 1-extendable colors exist for any graph G.

Theorem 2 *Let $G = (V, E)$ be an undirected graph, $n = |V|$. There exists a 1-extendable coloring of G with at most* $\min\{\lfloor 2\sqrt{n} \rfloor, \alpha(G)\}$ *colors.*

A naive approach to find a feasible solution of 1-EXTENDABLE COLORING consists in using heuristics for the classical COLORING problem, where we aim at partitioning the vertex set with the minimum number of independent sets. In this way, each channel is an independent set and is thus 1-extendable. We propose several more involved heuristics, based on Theorem 2, which consist in coloring the graph with various 1-extendable structures (independent sets, cliques, paths, bipartite graphs with a perfect matching,...). We compare these algorithms not only on unit disk graphs randomly generated but also on real-world data. We show that the heuristics proposed allow us to decrease significantly the number of channels of the network.

Références

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1. Exponential Time Hypothesis, see [2]