# Robust optimal sizing of water distribution networks facing intermittent demands

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## 1 Introduction

The problem is to equip with appropriate pipes a network whose topology is given and whose graph is a tree. Given the randomness of the demand, the problem falls under stochastic programming. However, the number of possible configurations of opening/closing faucets is exponential and would imply random binary variables. To overcome the obstacle, NeatWork in its early release [1] uses a heuristic for the design issue. The idea is to replace the actual flows with a deterministic approximation reflecting the average flow in each pipe as a function of the average number of open faucets downstream of that pipe and further adding a safety margin. This heuristic does not model reality because it does not reflect the essential condition of mass preservation, but it provides a basis for estimating and controlling the head losses in the pipes so as to ensure satisfactory flows at the faucets. Because the design is based on heuristic arguments, it is mandatory to couple the network sizing it generates with a Monte-Carlo simulation of real flows.

This paper discusses the design of a tree-shaped water distribution system for small, dispersed rural communities. It revisits the topic that was discussed in [2] and is nowadays implemented in the field [1]. It proposes a new approach to sizing by robust optimization to account for the uncertainty inherent in intermittent demands. It proposes a fast method of calculating stationary flows to test the performance of the networks thus designed by Monte-Carlo simulation.

## 2 The deterministic model for the design problem

The heuristic for the design relies on the following linear programming problem in which the flows on arcs $k$ are approximated by $\theta_k \bar{Q}$. $\bar{Q}$ is the target flow at all faucets and $\theta_k$ is a function of the average number of open faucets downstream of pipe $k$. The decision variable $x_{kd}$ is the fraction of the length $L_k$ of the arc $k$ that will be filled by the pipe $d$ of diameter $\Phi_d$ at the unit cost $C_d$. On each arc, one can use several pipes to satisfy physical conditions at a minimum cost.

$$\min_{x,a} \sum_{k \in N} \sum_{d=1}^{n} L_k C_d x_{kd} \tag{1a}$$

$$\sum_{k \in P_t} \sum_{d \in \mathcal{D}} \beta(\theta_k \bar{Q})^{\lambda} \frac{L_k}{\Phi_{kd}^{4.781}} x_{kd} + \frac{\bar{Q}^2}{\alpha} \quad \geq \quad h_0 - h_t, \ \forall t \in N_f \tag{1b}$$

$$\sum_{k\in P_t}\sum_{d\in\mathcal{D}}\beta(\theta_k\bar{Q})^\lambda\frac{L_k}{\Phi_{kd}^{4.781}}x_{kd} \quad\leq\quad h_0-h_t,\ \forall t\in N_b, \tag{1c}$$

$$\sum_{d\in\mathcal{D}}^{n} x_{kd} \quad=\quad 1,\ \forall k\in N \tag{1d}$$

$$s_t\geq 0\ \forall t\in N_f \text{ and } x_{kd} \quad\geq\quad 0,\ \forall k,d. \tag{1e}$$

$N_f$ is set of faucet nodes and $P_k$ is path from source to node $k\in N=N_f\cup N_b$. Constraint (1b) expresses the condition that head losses in the pipes and within the faucet itself match the driving force of gravity. The condition mimics reality because the actual flows upstream of the faucet are not known but just estimated at each node (and pipe incident to it) as a multiple of the target flow. The second term in the left is the impact of the faucet itself at the desired target flow. Constraint (1c) concerns intermediary nodes $t\in N_b$. It aims to protect against possible leakage. The objective promotes a choice of $x$ for the least cost equipment in each pipe. However, the cost of a pipe increases with its inner diameter, but lower diameters increase the head losses in the first sum of (1b) and (1c). The formulation by linear programming allows to find a compromise between the cost and the physical constraints. The parameters $\alpha$ and $\beta$ are physical coefficients.

# 3 Improved heuristic based on robust optimization

Given $n$ the total number of faucets downstream of arc $k$ and the random variable $\Theta_{p,n}$ of the number of taps open at its base following a binomial distribution $B(n,p)$, our experiments show that $[E(\Theta_{p,n}^2|\Theta_{p,n}\geq 1)]^{\frac{\lambda}{2}}\bar{Q}^\lambda=[\frac{np(q+np)}{1-q^n}]^{\frac{\lambda}{2}}\bar{Q}^\lambda$ is a sensible pointwise estimate of the traffic load through the pipe. But we must keep in mind that the true traffic is uncertain because of the stochastic behavior of the number $\Theta$ of open faucets. In the design phase, it is not possible to assess probabilities to the uncertain flow. Rather, it is possible to give tentative bounds of variation for $\Theta_{p,n}^\lambda$ for each individual pipes and use a robust optimization scheme to robustify constraints (1b).

Let $z_k=\Theta_k^\lambda$ and $y_k=\sum_{d\in\mathcal{D}}\frac{\beta L_k\bar{Q}^\lambda}{\Phi_{kd}^{4.781}}x_{k,d}$ be the coefficient of $x_{kd}$ in (1b). The linear constraint now reads

$$\sum_{k\in P_t} z_k y_k+\frac{\bar{Q}^2}{\alpha}\leq h_0-h_t,\quad \forall t\in N_f,$$

with an uncertain parameter $z_k$. Even if we had a perfect knowledge of the parameter $z_k$, it would hard to exploit it in the context of optimization. Robust optimization [3] proposes an alternative that has proved to be highly efficient. It is based on the use of a limited information on the the uncertainty that informally boils down to two ingredients : *i)* the coefficients $z_k$ can be assigned each an uncertainty interval and *ii)* not all $z_k$ can achieved their worst value simultaneously.

To assess the quality of the robust solutions, we perform Monte-Carlo simulations and solve a non-linear optimization problem to compute the flows at faucets. We propose a projected reduced Newton algorithm to calculate efficiently these stationary flows.

# Références


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