A Project and Lift Approach for a 2-Commodity Flow Relocation Model in a Time Expanded Network

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Introduction

Pickup and delivery problems covers a broad range of optimization problems and have been studied extensively due to its relation with real-world transportation tasks. A preliminary survey of these problems can be found in [1]. They can be classified in several subfamilies according with the nature of the requests involved (e.g., paired or unpaired demands), the location and number of depots (many-to-many, one-to-many-to-one, and one-to-one problems, etc.), or the type of constraints involved (time windows, capacities, transfers, etc.). In this work, we address a static variant of Pickup and Delivery with unpaired demands, transfers, and a time horizon (i.e., a global time window for performing the whole transportation process). Because this problem arises naturally in the context of items relocation to balance a system (e.g., bike-sharing systems [2]) it is sometimes called Relocation Problem [3].

Description of the Problem

We have a transit network connecting a set of stations, and those stations contain items of a homogeneous type (e.g., bicycles). According to an expected user demand, stations are classified into three categories: excess stations, deficit stations, and balanced stations. For each excess station, we determine the number of items that must be picked up from the station; symmetrically for each deficit station, we determine the number of items that must be delivered to the station. Balanced stations do not require to pick up or deliver items.

There is a homogeneous fleet of vehicles with finite capacity located at a depot station of the system. The fleet must perform a relocation process to "balance" the system within a given time horizon. This balancing operation consists in organizing a schedule for the fleet to pick up items from the excess stations and deliver them to deficit stations. Any station can be used temporarily to store items during the balancing process, and any vehicle can drop or take items at any station.

The aim of the problem is to compute a schedule for the fleet that minimizes a cost function consisting of a weighted sum of the costs related to the number of vehicles used, the distance traversed by the vehicles, and the time during which the items have been transported.

Model and Algorithms Proposed

We propose a "Project and Lift" approach for solving the Relocation problem. This approach is based on a 2-commodity flow model on a Time Expanded Network (TEN) for the Relocation Problem, that involves an integral flow vector which represents vehicles, and an integral flow vector which represents the items transported by those vehicles. Although the model is meaningful, and simultaneously copes with temporal and resource issues, it does not fit practical computation because it involves a large number of integer variables.

For getting a simpler model and continue taking profit of the powerful network flow machinery, we project the time expanded network model on the original transit network to obtain a 2-commodity flow "projected" model. This projected model only looks to “count” the numbers of vehicles and items that should pass through every arc of the original graph and hence it involves a significant smaller number of variables. In order to make this projected model compatible with the initial time expanded network model, we introduce a collection of Extended Subtour Constraints whose handling involves a separation process. We prove that this separation process can be performed in polynomial time, and discuss the implementation of a Branch-and-Cut algorithm. Then we enrich the model by adding a column generation procedure for separating a collection of Feasible Path Constraints related to item flow feasibility, and add this procedure as a part of the Branch-and-Cut algorithm.

The projected model can be very useful because, once we have found an optimal solution for a problem instance, we obtain a lower bound for the value of any optimal solution for the corresponding relocation problem, and we can also get an idea about which are the vertices and arcs that should be used by the vehicles to perform the relocation process. However, it turns out that even with all this information, it can be difficult to deduce a solution for the Relocation Problem. We call the "Lift" problem to the problem of constructing feasible solutions for the Relocation Problem starting from a solution of the Projected model. We distinguish several types of "lift" problems, according to their degrees of flexibility, and define an auxiliary graph which will be handy to set up mixed integer programming models for trying to solve those problems.

Results

We have considered a set of 20 problem instances with up to 100 vertices and 300 arcs. For the projected model, we have tested the Branch-and-Cut algorithm with/without Extended Subtour Constraints, and with/without Feasible Path Constraints, and we have examined its behavior both through solutions quality, and related running times. We have also constructed two MILP models for "lifting" solutions of the Projected Model: a "Strong" Lift model which is focused both on vehicle and item flow, and a "Weak" Lift model which is focused only on item flow. We observed the "Strong" model may give very good solutions, but usually is too exigent and results to be infeasible; in contrast, the "Weak" Lift model is more flexible and most of the time provide us with feasible solutions.

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