Parity Permutation Pattern Matching

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Mots-clés : *complexité parametrée, algorithmes.*

1 Introduction

Pattern matching for permutations, together with its many variants, has been widely studied in the literature [3]. More precisely, in the well-known problem Permutation Pattern Matching (PPM), given two permutations, a pattern σ and a text π, the task is to determine if π contains a subsequence which is order-isomorphic to σ. We introduce a natural variation of PPM, which we call Parity Permutation Pattern Matching, and that incorporates the additional constraint that the elements of σ have to map to elements of π with the same parity, i.e., even (resp. odd) elements of σ have to be mapped to even (resp. odd) elements of π.

2 Results

We study the classical and the parameterized complexity of Parity Permutation Pattern Matching. A summary of the results can be seen in Table 1.

2.1 Parameterized complexity

While it is known that Permutation Pattern Matching is in FPT [4], we show that adding the parity constraint to the problem makes it W[1]-hard parameterized by the length of the pattern, even for alternating permutations or for 4321-avoiding patterns.

The approach used to prove that PPM is FPT is based on a result that states that given a permutation π, there exists a polynomial time algorithm that either finds an $r \times r$-grid of π or determines that the permutation has bounded width (and then returns a decomposition, which is used to solve the PPM problem in FPT time). This win-win approach works because, if π contains an $r \times r$-grid, it's not hard to see that it contains every possible pattern σ. However, this cannot be generalized to Parity PPM, as here we have no information on the parity of the elements of the grid, and thus, it is not guaranteed that every pattern maps via a parity respecting embedding into the grid. We give a parameterized reduction from k-Clique to show that the problem is in fact $W[1]$-hard parameterized by the length of the pattern.

On the other hand, Parity PPM remains in FPT if the text avoids a fixed permutation, as then we can make use of the twin-width framework.

2.2 Classical complexity

With respect to the classical complexity, Parity Permutation Pattern Matching remains polynomial-time solvable when both permutations are separable, if both are 321-avoiding, or if both are $(231, 213)$-avoiding, as for PPM [2, 5, 1, 7], but NP-hard if the pattern is 321-avoiding and the text is 4321-avoiding, which is also the case for PPM [6].

	PPM	Parity PPM
General case	NP-hard, FPT	W[1]-hard
Separable permutations	P	P
(231,213)-av σ and (231,213)-av π	P	P
321-av σ and 321-av π	P	P
321-av σ and 4321-av π	NP-hard	NP-hard
4321-av σ	FPT	W[1]-hard
Alternating π and σ	FPT	W[1]-hard
π is fixed pattern avoiding	FPT	FPT

TAB. 1 – Summary of known results (for PPM) and our results (for Parity PPM).

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