Quantum Solutions to Job Shop Scheduling Problems

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1 Introduction

One of the most promising field for quantum computing is quantum optimization, that is the use of quantum computers (QCs) and quantum solutions for solving complex optimization problems. Among the approaches developed for combinatorial optimization are exact methods derived from the Grover search algorithm [4], and meta-heuristics, such as the Quantum Approximate Optimization Algorithm (QAOA) [3] and the Quantum Annealing [5]. Exact methods and the QAOA are developed for universal gate-based QC, whereas quantum annealing is designed for analogic quantum computers, namely developed by D-Wave. Solving a combinatorial optimization problem with heuristics requires a transformation of the problem to a format suitable for the QC. While some existing frameworks[1] can cope with Constrained Quadratic Models (CQM), Quadratic Unconstrained Binary Optimization (QUBO) is the best option to map an optimization problem to a QC or a quantum simulator. We consider in this paper the job shop scheduling problem. We review the quantum-based methods recently proposed in the literature to solve the problem. In particular, we focus on mathematical formulations that can be easily adapted to cope with constraints that are relevant in practice. The related QUBOs will be detailed at the workshop.

2 Problem definition

The job shop scheduling problem can be stated as follows: A set of n jobs $J = \{J_1, J_2, \dots, J_n\}$ has to be processed on a set of m machines $M = \{M_1, M_2, \dots, M_m\}$. Each job J_i consists in a linear sequence of n_i operations $(O_{i1}, O_{i2}, \dots, O_{in_i})$. Each machine can process only one operation at a time and each operation O_{ij} with a processing time of p_{ij} time units needs only one machine. Each job visits the machines according to its own predefined routing. This problem generalizes the flow shop scheduling problem, in which all the jobs are processed following the same routing $(M_1, M_2, \dots, M_m)$. The objective is to determine the starting date t_{ij} of each operation O_{ij} so that the makespan noted C_{max} is minimized.

3 Quantum solutions

While the scientific literature on quantum solutions for hard combinatorial optimization problems is becoming important and diversified, studies of shop scheduling problems are still scarce. In [8] the authors proposed a quantum annealing solution for the job shop scheduling problem with the makespan objective. The method was implemented on a D-wave quantum annealer. Recently, the authors of [1] have proposed four variational quantum heuristics for solving a job shop scheduling problem with early and late delivery as well as production costs, adapted from a steel manufacturing process. They have compared the performance of the heuristics on

[1] https ://cloud.dwavesys.com/leap

two-machine instances using IBM quantum processors. Using the same QUBO formulation as [8], authors in [6] proposed a hybrid quantum annealing heuristic to solve a particular instance of the job shop scheduling problem on the D-Wave 2000Q quantum annealing system. A generalization of the job shop scheduling problem with pools of parallel machines available for processing operations was considered in [2]. The authors proposed a QUBO derived from the one of [8] and an iterative procedure to solve relatively large size instances. Another QUBO formulation is proposed in [7] for assigning dispatching rules to the machines and scheduling the operations in a flexible job shop system. The problem is solved using the leap hybrid solver. With a hybrid approach that combines constraints programming and a QUBO model, the autors in [9] solve efficiently large instances of the job shop scheduling problem on a specialized hardware[2].

4 Conclusions and perspectives

We review the recent solutions proposed to solve the job shop scheduling problem on quantum computers, with a focus on QUBO formulations that help integrating relevant constraints. While the existing hardware are still limited in their ability to handle the number of variables generated, it remains important to progress on the modeling of practical problems. Future works include investigating mechanisms for keeping the number of variables low while integrating new relevant constraints.

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