Optimal bus scheduling to minimize passengers delay

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keywords: Public transport scheduling, Stop-Skipping, Np-Hard, Optimisation, Delay

1 Introduction

The public transport system is considered as the backbone of sustainable urban development, since it allows more efficient movements in cities. However, this system is sensitive to traffic congestion, weather conditions, and unstable demand patterns which lead to uncomfortable travel time for both the passengers and the operators. Solutions such as the increase of the frequency of the bus lines and bus control strategies, i.e dedicated bus lanes and signals, vehicle holding, stop-skipping and deadheading are not enough to improve the efficiency and the reliability of the bus systems [2].

In today’s situation, buses of the same line stop at all stations forming a schedule of served stations. The stop-skipping (also known as expressing, or limited-stop service) is a control measure that allows a vehicle to skip a stop (or a series of stops) of the same line if it is running behind schedule [1]. To provide a resilient and a dynamic service, we define a strategy similar to the stop skipping that allows buses of the same line to skip some stations in order to minimize the time until the last passenger reaches his/her destination, which is the delay. The purpose is to decide at the beginning of each turn which stations to be served by each bus that leads to the minimization of the delay. This decision can change from one turn to another and must be made based on previous knowledge of traffic and demand of passengers.

2 Model formulation

Consider a first theoretical simplified model of the public transport system represented as a ring $R$ made of $N$ consecutive cyclic slots numbered from $0$ to $N-1$. $M$ buses serve $K$ stations where each serving bus $B_i$ for $1 \leq i \leq M$ has a given starting and ending station on the ring, and a serving vector $D_i$ (see Figure 1). Each serving vector $D_i$ indicates the stations served by bus $B_i$ such that $D_i[S_j] = 1$ means the bus will stop at station $S_j$, 0 otherwise. Each slot on the ring represents the position of buses and stations, and helps define the next bus’s movements. Each slot can be occupied by only one bus at once.

The proposed model is based on the following assumptions: (1) the origin-destination demand matrix is given at the beginning, specifying the number of passengers waiting at each station at time step $\tau = 0$ and having a specific destination. (2) Passengers board the first arriving bus serving their destination without interconnection while respecting the bus capacity constraint. (3) Passengers waiting at a station with different destinations board based on a uniform distribution. (4) The buses stop for one time step only for passengers to board and alight. If a bus is stopped at a station and a following bus wants to stop at this station, the following bus will be blocked on the ring. (5) For each bus, the starting and ending stations of the route can be any station on the ring. (6) The route is a one direction route with no passengers alighting at first station ($S_1$) and no passengers boarding at last station ($S_K$).
FIG. 1: Model architecture of the transport system

3 Problem

Given the number of buses, the number of stations, and the demand origin-destination matrix, the goal is to find a schedule that serves all passengers waiting at stops with minimum expected delay. We show that the problem is NP-hard and we conjecture it is not in NP (if P ≠ NP) because it is not possible to anticipate in polynomial time which buses will be blocked and on which slots. Thus, it seems difficult to be solved by exact algorithms even for small instances.

Calculating the real delay value is time consuming considering the large number of constraints in our model to be evaluated at every time step. Thus, we define a new parameter (load-delay) and prove that the problem of minimizing the load-delay gives a rise to a problem in NP for any schedule of any instance of this problem. Numerical experiments show the correlation between the real expected delay and the load-delay for all schedules of the same problem instance and that the minimization of the load-delay problem is NP-complete which allows us to use meta-heuristics.

Thus, the Simulated Annealing meta-heuristic is used and we compare its results and performance with the original problem of minimizing the real delay. We also propose a reinforcement learning approach as a game model to be tested and compare its results to the Simulated Annealing results. Because of the page number constraint, these results and further details will be presented throughout the congress.

4 Conclusion and perspectives

The proposed meta-heuristic is the first step to solve the proposed static model. This model is chosen because it can be developed in the future to be used in a more dynamic context with varying real time demand and road conditions. In dynamic context, the stop skipping strategy is best implemented with autonomous buses that can change their stopping strategy in real time to adapt to the before mentioned variations. This approach can be validated with reinforcement learning techniques.

References
