The Storage Location Assignment and Picker Routing Problem : A Branch-Cut-and-Price Algorithm

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1 Introduction

Warehousing logistics is one of the key aspects of the performance of a supply chain. The Order Picking (OP) activity, i.e. the action of retrieving products from their storage locations, is often considered the most resource-intensive operation in the warehouse, and the main lever of performance enhancement [1]. The recent development of the e-commerce practice has increased the need for an agile and responsive system, to be able to guarantee a high level of responsiveness to a very dynamic context. In this situation, a shift of paradigm has appeared in the organizational methods for warehousing logistics. In this talk, we study the integration of two decisions (the storage and the picker routing) that are classically considered at two separate levels of decision (tactic and operational). However, new industrial practices have challenged this assumption, as storage decisions are becoming more and more dynamic.

2 Problem Description and Related Literature

Two of the main decision problems encountered in OP are the Storage Location Assignment Problem (SLAP) and the Picker Routing Problem (PRP). The SLAP deals with the assignment of the different products to the storage locations, with the aim of improving the efficiency of the OP. The PRP is the main problem that takes place during OP : the aim is to route the picker through the warehouse to retrieve all the items of an order or a batch of orders. Both problems have attracted plenty of attention from the literature [1], with a large variety of variants and methods.

In this talk, we study an integrated version of these two problems, in the Storage Location Assignment and Picker Routing Problem (SLAPRP) the two levels of decision are jointly optimized. This integration is natural, as both problems are strongly linked. Indeed, an assignment plan is needed to solve a PRP instance, and the only way to assess exactly the performances of a SLAP is to compute the traveled distance of the picker, i.e. to compute, or estimate, the solution of a PRP. It is therefore not surprising that the two problems have already been studied together [3], yet most of the time the two problems are solved sequentially and few works study the integrated version. These works include a study of the problem complexity, which is NP-hard in the strong sense, even in simple cases [2]. The work of [4] studies the SLAPRP and proposes MIP formulations for several variants of the picker routing policy, as well as a metaheuristic method. The authors propose a set of benchmark instances that we use to evaluate our algorithm.

The contribution of this work is two-fold : first, a structural analysis of the problem led to the introduction of a novel extended formulation for a large class of variants of the SLAPRP, especially with different picker routing policies, as well as the introduction of a family of non-robust valid inequalities. Secondly, we propose a competitive solution method derived from the integration of this formulation in a Branch-Cut-and-Price framework.

3 Solution Method

We introduce a novel extended formulation for the SLAPRP based on a Dantzig-Wolf reformulation. With the extended formulation, all the practical details of the problem (e.g., warehouse layout, routing policy) are convexified into the subproblems, leading to a very generic formulation that fits a large variety of operational problems. As such, we propose a generic solution method that is able to solve different variants of the SLAPRP.

Compared to the compact formulation, the extended one provides a significant improvement in the dual bound. However, the number of variables becomes exponential and the formulation needs to be solved by column generation to remain tractable. The resulting pricing problems can be modeled as variants of the Elementary Shortest Path Problem with Resource Constraints (ESPPRC), solved by a classical label-correcting algorithm. The details of the operational configuration at hand are convexified, and as such, they have an impact on the pricing problem. The backbone of the algorithm remains identical among all configurations, but some changes need to be considered in the underlying graph and the resource extension functions.

The dual bound is further improved by the addition of a novel family of valid inequalities called Strengthened Linking (SL). The SL inequalities enforce a stronger link between the storage and routing aspects, providing a significant improvement in the dual bound. However the family of SL inequalities is exponential in size, and the cuts are non-robust with the formulation. In this case, particular attention is paid to the management of such cuts, to keep their proliferation under tight control. The column and cut generation is then embedded in a Branch-Cut-and-Price framework.

4 Results and Conclusion

The algorithm presented in this talk is compared to other approaches on various sets of benchmark instances of the literature [4]. The Branch-Cut-and-Price outperforms other approaches on most configurations, and is able to solve to optimality a large number of previously open instances. Unsurprisingly, the algorithm underperforms compared to commercial solvers with the most simple configurations (e.g. S-shape routing), where the problem is structurally much simpler, giving an edge to a dedicated compact formulation. Managerial insights are also derived from the computational experiments.

Références

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