

# A column generation approach to solve the Joint Order Batching and Picker Routing Problem including congestion

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## 1 Introduction

Order picking is one of the most important processes in the context of warehouse operations and concentrates the majority of the operational costs. Picking process consists in the preparation of a given set of customer orders. The large investments costs to built an automated system, and the existence of a highly fluctuating demand, keep human-operated systems popular. Picker-to-parts systems, that consists in pickers navigating around the warehouse to collect all the requested orders, represent the majority in warehouses.

The aforementioned picker-to-parts systems necessarily considers the resolution of the picker routing problem (PRP). The PRP is to determine the sequence (or route) in which a set of products will be picked, minimizing the total distance or time. The route should start and finish in a given point named depot. The PRP can be solved only if the set of products to pick has already been defined. The order batching problem (OBP) is to decide how the customer orders are grouped (or batched) together respecting the trolley capacity for each batch. The objective of the OBP is to find a set of batches that minimizes the total distance or time. To solve the OBP, it is necessary to consider a fixed routing policy. When considering the optimal routing policy, the OBP and the PRP are integrated in a joint model [2] named Joint Order Batching and Picker Routing Problem (JOBPRP). In this work, we propose to model the delay produced by picker congestion, and provide an extended Mixed Integer Linear Program (MIP) formulation for the JOBPRP with picker congestion. This formulation is solved using a column generation approach.

## 2 Modelling congestion into the warehouse

In the literature, a common assumption is the non-existence of congestion produced by the pickers. It is assumed that all the pickers are performing their tour independently whereas it is clearly not the case in practice since all the pickers are walking in the same picking area. When several pickers are in the same space at the same time, this produces congestion that implies delay for the pickers. In the warehouse, congestion has consequences on costs and performance [1]. In this work we consider the operations in a rectangular warehouse with parallel vertical and cross aisles. Vertical aisles contain racks in which the products are located and cross aisles allow to move between vertical aisles. We consider a sub-aisle (intersection between a vertical-aisle and the space between two consecutive cross-aisles) as the physical reference to compute the delay produced by congestion. Due to the characteristics of the human behavior, it is not appropriate to generate an exact coordination of the different pickers. Thus, we propose a

rough estimation of the level of congestion by introducing a time discretization that divides the planning horizon of the order picking process into homogeneous time intervals. For each time period, based on the number of pickers in each sub-aisle, an extra time given by a delay function is imposed to all the pickers in that sub-aisle. Note that computing congestion is typically non linear since the delay produced in one time period will generate an additional delay in the future. Include congestion and minimize the total time can produce undesirable situations. For example it is possible to avoid congestion by waiting in the extreme of a sub-aisle or following inefficient paths (with no practical sense for the picker). Thus, it is relevant to define that waiting time is not allowed and to limit the routes possibilities in the warehouse navigation.

### 3 A column generation approach

We propose a general extended formulation to solve the JOBPRP including the effect of congestion, that minimizes the total completion time. Each column represents a picking route with the associated timing of operations, and the level of congestion of the visited sub-aisles at the corresponding time periods. The model selects only compatible routes in terms of level of congestion in each sub-aisle and time period. To obtain lower bounds, we solve the linear relaxation of the extended formulation with a column generation algorithm. In each iteration, a dedicated dynamic programming approach is called to provide negative reduced cost columns. The dynamic programming is solved with a labelling algorithm that simultaneously explores routes and levels of congestion. The main interest of that algorithm is to easily handle the defined routing policies and manage the non-linear characteristics of the congestion. In order to obtain a performance guarantee, in each iteration of the algorithm, a Lagrangian bound is computed. Finally, to obtain an upper bound, the extended formulation is solved with the columns obtained during the column generation procedure.

### 4 Conclusions

In this work we analyze the effect of congestion in picker operations, integrating this concept to the JOBPRP. We propose a modelling of the delay produced by congestion and define relevant routing policies. A general extended MIP formulation is presented and solved by a column generation heuristic. A dedicated dynamic programming algorithm is proposed to solve the pricing problem. Preliminary results on small size instances show that we are able to solve the linear relaxation to optimality in reasonable time. Additional computational results will be presented and discussed.

### Références

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