What is the optimal cutoff grade for multiple minerals ?

Bernardo Pagnoncelli[1], Adriana Piazza[2], Lewis Ntaimo[3]

[1] SKEMA Business School Université Côte d'Azur, France
bernardo.pagnoncelli@skema.edu

[2] Department of Economics, Faculty of Economics and Business, University of Chile
apiazza@fen.uchile.cl

[3] Wm Michael Barnes '64 Department of Industrial & Systems Engineering, Texas A&M University
ntaimo@tamu.edu

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1 Introduction

The standard practice in mine planning is to first define the contour of a mine by solving the ultimate pit problem, and then to perform a cutoff grade optimization to define at each moment in time which material should be mined and processed ; the rest is considered waste. The seminal work of K. Lane [1] established a unified framework to perform cutoff grade optimization, taking into account economic factors, production capacities, and the time value of money. The algorithm proposed in [1] is widely used in commercial software for the mining industry, and its optimality has been characterized by [2].

We study deposits with multiple minerals with special attention to the two-dimensional case. Mines that contain more than one economic mineral include copper-gold, lead-zinc, copper-lead-zinc, among others. Our main contribution is to show that, in two dimensions, the solution that maximizes the operations' net present value is formed by grade-pairs that can be defined as the region above a line. We also show that in n dimensions the optimal cutoff surface is a hyperplane.

2 The mining scheduling problem

We consider the mining operation as a succession of three stages : mining, concentrating, and refining. We say that a function $\lambda : [0, \bar{g}_1] \times [0, \bar{g}_2] \to [0, 1]$ is a grade density function describing an homogeneous orebody if it is Lebesgue-integrable with $\int_0^{\bar{g}_1} \int_0^{\bar{g}_2} \lambda(g_1, g_2) dg_1 dg_2 = 1$, where $\bar{g}_i$ is the highest grade of mineral i present in the mine. We define *set of admissible grade-pairs*, denoted by $\Omega \subseteq [0, \bar{g}_1] \times [0, \bar{g}_2]$, as the set of grade-pairs $(g_1, g_2) \in \Omega$ that is sent to the concentrator ; the rest is waste. Figure 1 shows some of the level sets of the grade density function λ together with the set Ω in gray.

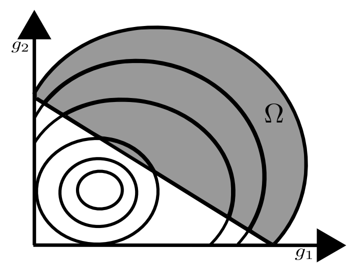

FIG. 1 – A cutoff line, with admissible grade-pairs in gray.

We define $Q_{m,t}$ as the amount of material to be extracted from the deposit at time t, $Q_{c,t}$ is the amount of extracted material to be sent to the concentrator, and $Q^1_{r,t}$ and $Q^2_{r,t}$ are the amount of minerals 1 and 2 to be refined, respectively. They can be characterized as

$$Q_{c,t} = Q_{m,t} \iint_{\Omega_t} \lambda(g_1, g_2) dg_1 dg_2, \quad Q^i_{r,t} = Q_{m,t} z_i \iint_{\Omega_t} g_i \lambda(g_1, g_2) dg_1 dg_2, \quad i = 1, 2,$$

where z_i is the recovery rate. We can write the mining schedule problem in two dimensions as

$$V(U_o) = \begin{cases} \max\limits_{\Omega, Q_m, Q_c, Q_r} & \sum_{t=0}^{T} \delta^t [b_1 Q^1_{r,t} + b_2 Q^2_{r,t} - c Q_{c,t} - m Q_{m,t} - f w_t] \\ s.a. & Q_{c,t} \leq w_t C, \\ & Q^i_{r,t} \leq w_t R_i, \qquad \text{for } i = 1, 2, \\ & Q_{m,t} \leq w_t M, \\ & w_t \in [0, 1], \\ & \sum_{t=1}^{T} Q_{m,t} \leq U_o, \end{cases} \tag{1}$$

where (M, C, R_i) and (m, c, r_i) are upper limits and unit costs, respectively, in the mining operation, s_i is the sale price, f is a fixed cost, d is the discount rate, $\delta = 1/(1 + d)$, T is the time horizon, w_t is the percentage of the time period over which the mine is operational, and U_0 is the material left to be extracted.

2.1 Bilevel reformulation of problem (1)

$$V(U) = \max_{Q_m \in [0, \min\{wM, U\}],\ w \in [0,1]} \{v(Q_m, w) + \delta V(U - Q_m)\}, \quad \text{where}$$

$$v(Q_m, w) = Q_m \max_{\Omega} \left\{ \iint_{\Omega} (b_1 g_1 + b_2 g_2 - c) \lambda(g_1, g_2) dg_1 dg_2 \right\} - m Q_m - f w$$

$$s.t. \iint_{\Omega} \lambda(g_1, g_2) dg_1 dg_2 \leq wC/Q_m, \quad z_i \iint_{\Omega} g_i \lambda(g_1, g_2) dg_1 dg_2 \leq w R_i / Q_m, i = 1, 2.$$

2.2 Theorem

The optimal curve Ω is a line in 2-D, and a hyperplane in n-dimensions.
Idea of the proof : We use Green's Theorem to convert integrals of areas into line integrals, and then Euler-Lagrange equations to derive the necessary conditions the curve has to satisfy. Considering all possible active constraint cases, we concluded the optimal curve is a line. In [3] we extended the results for the n-dimensional case using the generalized Stokes Theorem and showed that the optimal solution is a hyperplane.

Références

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