A matheuristic for the 2D bounded-size cutting stock problem

Alexis Robbes[1], Khadija Hadj Salem[2]

[1] Université de Sorbonne, LIP6, 4 place Jussieu, Paris Cedex 05, 75252, France.
alexis.robbes@lip6.fr

[2] Mines Saint-Etienne, Univ Clermont Auvergne, CNRS , UMR 6158 LIMOS, Institut Henri Fayol, F-42023 Saint-Étienne, France
khadija.hadjsalem@emse.fr

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1 Problem definition

In this paper, we consider a variant of the *two-dimensional Cutting Stock Problem* (2D-CSP). First introduced in 1961 by [1], the 2D-CSP consists of cutting small 2D items in larges plates. Often the 2D-CSP is considered with a fixed number of guillotine cuts to produce rectangular items. Even with these two constraints, the 2D-CSP is $\mathcal{NP}$-hard. In the typology of [5], a CSP with a strongly heterogeneous assortment of plates is named a *Residual Cutting Stock Problem* as in practice the heterogeneity is due to a previous cutting step. However, some factories can manufacture requested plates with highly customizable sizes using adjustable molds. For this type of production, the plate sizes are bounded and the cutting objective is to minimize the total production. We call this variant as the *two-dimensional Bounded-Size Cutting Stock Problem* (2D-BSCSP).

More formally, we are given a set of rectangular items I, where each item i has a width w_i, a height h_i, and a demand d_i. These items must be cut in a two-stage guillotine manner from plates of width in $[W^{LB}, W^{UB}]$ and height in $[H^{LB}, H^{UB}]$. A manual trimming step is allowed to separate items and waste parts. The objective is to find a cutting plan with the minimum sum of plate area.

An illustrative example of the 2D-BSCSP is given in Figure 1, where a set $\mathcal{I} = \{1, 2, 3\}$ of item types has to be cut from plates with bounded sizes. The feasible solution contains only one pattern i.e. a cutting plan on one plate. This pattern has a width of $2 \cdot w_1$ and height of H^{LB}. Three types of waste are produced, ▤ waste produced by first stage guillotine cut, ▥ waste produced by second stage guillotine cut, and ▨ waste produced by handmade trimming.

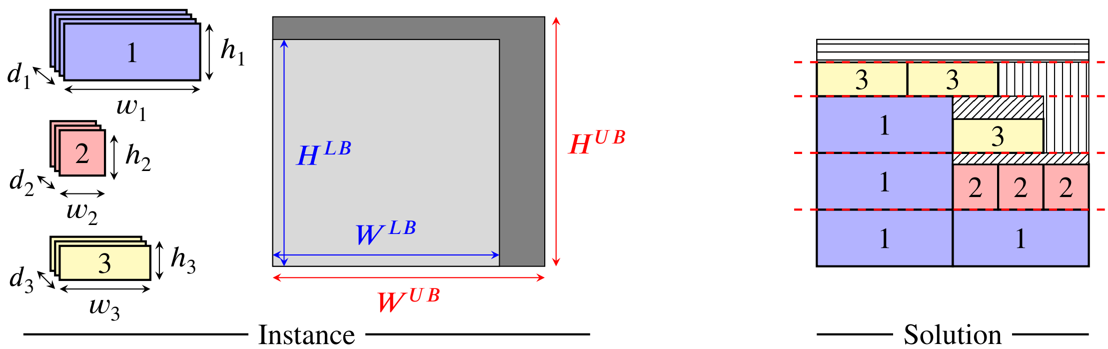

FIG. 1 – An instance of 2D-BSCSP and a feasible solution

2 A strip-based matheuristic

2D-CSPs were tackled through various approaches, mainly based on generating cutting patterns. Just a few methods were considered for the concept of strips, e.g. in [3]. Several approximate algorithms start by generating strips, filling them with pieces, and subsequently searching for the "best" combination of these strips. More recently in [4], a strip-based compact formulation for guillotine 2D-CSPs is proposed, in which a set of two decision variables is considered to define the position of cut of each item type.

To solve the 2D-BSCSP, we first propose a new *strip-based MILP* formulation, where a binary variable represents if a strip is in the same pattern as another one. The formulation is inspired by a recent modeling of the Bin Packing problem proposed in [2]. As the sizes of the plates are bounded, we also consider one continuous variable per strip which represents the wasted height of the pattern defined by this strip.

We then propose a matheuristic based on our *strip-based MILP* formulation. The general flowchart of the proposed matheuristic is shown in Figure 2. In the first step, the set of all possible strip type Θ (respecting W^{LB} and W^{UB}) is generated, then a relaxed version of the problem where $H^{LB} = 0$ is quickly solved with an ILP. With the resulting strip selection $\mathcal{S}$, a compatibility matrix $(c_{\theta\sigma})$ is computed. The compatibility between two strip types θ and σ is based on the gap between the minimum waste pattern containing θ and the minimum waste pattern containing θ and σ together (restrain on $\mathcal{S}$). Thanks to a threshold parameter, the weakest compatibilities are ignored and a sparse binary compatibility matrix $(\delta_{\theta\sigma})$ is constructed. To obtain a feasible heuristic the instance is solved with our *strip-based MILP* restrain on the subset of strip $\mathcal{S}$ and the sparse compatibility matrix $(\delta_{\theta\sigma})$. As the $\mathcal{S}$ is not necessarily the optimal strip selection, a loop removes the less compatible strip type computed ($\arg\min_{\sigma}(\sum_{\theta} c_{\theta\sigma})$) and the less gainful strip type (i.e. the strip type with the worst use ratio in the best solution).

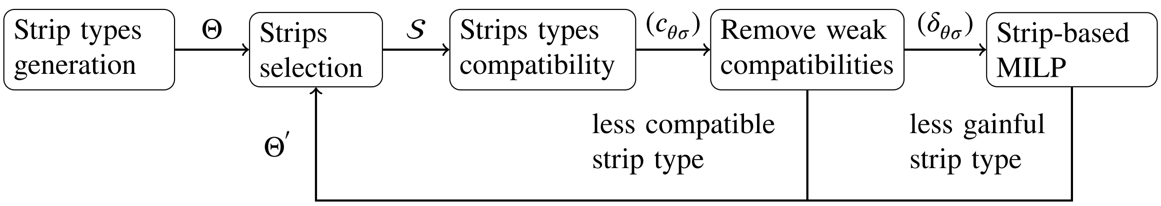

FIG. 2 – An instance of 2D-BSCSP and a feasible solution

We implemented the proposed approaches and conducted our experiments using two sets of instances from the literature, available at
http ://or.dei.unibo.it/library/2dpacklib. Numerical results will be given and analyzed during the presentation.

Références

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