Robust optimization applied to glass production

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Context

In the glass industry, visual and thermal properties of the glass sheets are obtained via the deposit of very thin layers of different materials. A standard way to perform this step is the use of a "magnetron," in which the materials are transferred from cathodes to the sheets using a magnetic field. Different materials can achieve a given property of the glass. Since the cathodes are very expensive, their activation and replacement have to be carefully decided to keep the production costs and the waste of materials low. Due to the complexity of the physical process, the consumption of the cathodes is partly uncertain. Moreover some urgent orders may be added, changing the initial production plan. All of it makes finding the best activation and replacement decisions a highly challenging task. In this process, the production is organized in campaigns containing several tens of orders and separated by shutdowns. In these periods, maintenance of the magnetron is conducted based on the activation and replacement decisions. By consequence, in an accurate modeling, the decisions have to be sequentially taken, leading to a multi-stage robust optimization problem. A timeline of decisions and uncertainty realizations is provided on figure 1.

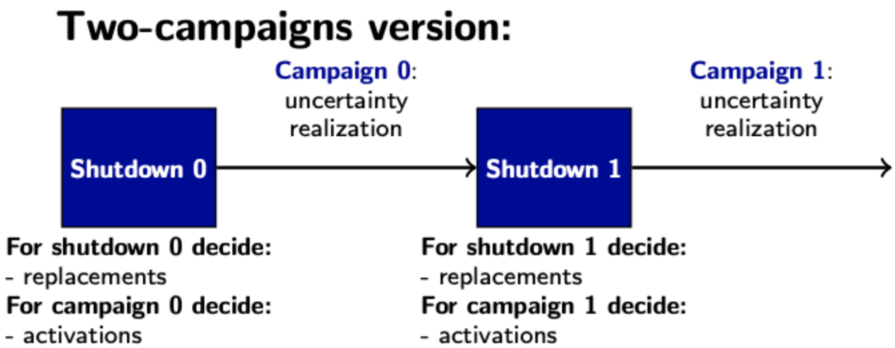

FIG. 1 – Decisions and uncertainty realizations timeline

Contributions

To tackle the over-conservativeness of the current industrial practice, we propose to solve this problem within the framework of finite adaptability, introduced by Bertsimas and Caramanis [1]. It consists in splitting the uncertainty set into finitely many parts and in assigning to each part a constant recourse decision. Let us denote by x the "here and now" and y the "wait and see" variables. The constraint matrices A and B linearly depend on an uncertain parameter $\omega \in \Omega$, where Ω is the uncertainty set. The K-adaptability two-stage problem is

$$\begin{array}{lll} \min\limits_{\substack{\Omega_1 \cup \cdots \cup \Omega_K = \Omega \\ x, y_1, \ldots, y_K}} & c^\top x + \max\left\{d^\top y_1, \ldots, d^\top y_K\right\} & \\ \text{subject to} & A(\omega)x + B(\omega)y_1 \leqslant e & \forall \omega \in \Omega_1 \\ & \vdots & \vdots \\ & A(\omega)x + B(\omega)y_K \leqslant e & \forall \omega \in \Omega_K\,. \end{array}$$

For the two-stage version of the studied problem, the "here and now" variables correspond to the decisions which have to be taken during the first shutdown, and the "wait and see" variables partly correspond to the cathodes' consumptions during the first campaign and replacement and activation decisions of the second shutdown.

Introduced in 2010, some heuristic methods were proposed to solve the K-adaptability problem (see [2] and [4]). Recently, Subramanyam et al. [5] proposed an exact method able to solve efficiently finite adaptable two-stage robust problems.

First, we show how to reformulate the studied problem in its two-stage version within the setting of Subramanyam et al. using a duality trick [3]. Preliminary results clearly demonstrate that finite adaptability brings a significant improvement upon the non-adaptable robust solutions, which are already better than the industrial solutions. Second, to deal with the full problem, we propose a multi-stage extension of the method by Subramanyam et al. Along with this extension, we describe a possible way to improve the lower bounds used within this exact method, as well as an heuristic for computing quickly feasible solutions. The theoretical and experimental validation of these extensions is currently under investigation. Third, we reformulate the K-adaptability model using affine adaptability for continuous "wait and see" variables and study the impact on the performance of the solving approach. Bertsimas and Caramanis have given evidences that in many situations finite adaptability provides a good approximation to the "complete" adaptability. Our last contribution is a theorem in the line of their work, showing that finite adaptability approximates asymptotically complete adaptability under mild continuity conditions.

Références

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