Branch-and-Price for a Concurrent Open Shop Problem

Michele Barbato[1], Diego Delle Donne[2], Emiliano Lancini[3]

[1] Dipartimento di Informatica, Università degli Studi di Milano, Milan, Italy
michele.barbato@unimi.it
[2] ESSEC Business School, av B. Hirsch, 95021 Cergy-Pontoise, France
delledonne@essec.edu
[3] Université Paris Dauphine - PSL, LAMSADE, CNRS, UMR 7243, Paris, France
emiliano.lancini@lamsade.dauphine.fr

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1 Introduction

In this work, we present some preliminary results of a study on a variant of the Open Shop Scheduling Problem. We are given a time horizon and a set of jobs, each composed of several tasks with unitary duration. Each task must be processed on a specific machine. Each machine can process at most one task at a time. The tasks of a given job can be processed in any order, and possibly two or more tasks of a same job can be processed at the same time. The problem consists in finding a schedule of the tasks on the machines, within the time horizon, that minimizes the average number of time slots used by each job. We call this problem the *Concurrent Open Shop Coloring Problem* (COSC).

Model. To tackle this problem, we present a new edge-coloring model.

Let $B = (M, P; E)$ be a bipartite graph, with shores P and M, and let k be an integer value. The resolution of the COSC requires to assign a color form 1 to k to each of the edges of B so that for every vertex $w \in P$, all edges incident to w are assigned to distinct colors. This coloration has as objective the minimization of the average number of colors assigned to edges incident to each of the vertices of M.

Related Problems. Research on concurrent open shops spans over a wide range of different applications and, as a consequence, a wide range of names. Different approaches motivate different objective functions, the most studied being the average weighted completion time [5]. We refer to the recent monograph of Kubiak [4] on Open Shop Problems for a more detailed analysis of the literature. The parallelism between scheduling and coloring is well known and studied in the literature. In particular, there is a rising interest in coloring models for concurrent open shop, as testified by the work of Grinshpoun et al. [2, 3]. At the best of our knowledge, the COSC has not been proposed in the literature. However, we can see an analogy between this problem and the Maximum Impact Coloring Problem of Braga et al. [1].

2 A Covering Formulation

One can prove that the COSC is **NP**-Hard. Therefore, we implemented a covering formulation which we solve with an exact procedure. Here, $\mathcal{A}$ is the collection of all subsets A of E such that no two edges in A are incident to a same vertex in P. Binary variable x_A states that all the edges in A receive the same color.

$$\min \sum_{A \in \mathcal{A}} M_A x_A \quad \text{s.t.} \tag{1}$$

$$\sum_{A: e \in A} x_A \geq 1 \qquad \forall e \in E, \tag{2}$$

$$\sum_{A \in \mathcal{A}} x_A \leq k \tag{3}$$

$$x_A \in \{0,1\} \qquad \forall A \in \mathcal{A}, \tag{4}$$

where for each $A \in \mathcal{A}$, M_A is the number of vertices of M covered by A. Clearly, this formulation uses an exponential number of variables.

3 A Branch-and-Price Procedure

We developed a branch-and-price procedure for COSC. To solve the linear relaxation of the model, the pricing problem consists in a variant of the star packing problem. Given a value associated with the edges of B, the pricing problem consists in finding a set of vertex-disjoint stars rooted in M for which the sum of the values of the edges minus the number of stars in the set is smaller than a given value γ issue from the duel values.

We developed a branching scheme following a Ryan-Foster branching rule. From a pair of edges of E, we create two subproblems: in one of these, the two edges are forced to receive the same color while in the other they are forced to receive different colors. We prove that this procedure completely explore the space of integer solutions.

We present the computational results of our procedure on a set of artificial instances. Those instances vary from toy problems to medium-sized instances.

References

[1] Mónica Braga, Diego Delle Donne, Rodrigo Linfati, Javier Marenco. The maximum-impact coloring polytope. *International Transactions in Operational Research*, 24:303–324, 2017.

[2] Tal Grinshpoun, Hagai Ilani, and Elad Shufan. The representation of partially-concurrent open shop problems. *Annals of Operations Research*, 252(2):455–469, 2017.

[3] Hagai Ilani, Tal Grinshpoun, and Elad Shufan. Bounded colouring motivated by the limited resource partially concurrent open shop problem. *Annals of Operations Research*, 302(2):461–476, 2021.

[4] Wieslaw Kubiak. *A Book of Open Shop Scheduling: Algorithms, Complexity and Applications*, volume 325 of *International Series in Operations Research & Management Science*. Springer International Publishing, Cham, 2022.

[5] Thomas Roemer. A note on the complexity of the concurrent open shop problem. *Journal of Scheduling*, 9(4):389–396, 2006.