Semi-Amortized Models for Lagrangian Relaxation

F. Demelas*, M. Lacroix*, J. Le Roux*, A. Parmentier°

*LIPN, Université Sorbonne Paris Nord — CNRS
°CERMICS, École Nationale des Ponts et Chaussées

Summary We present machine learning techniques to predict parameters of Lagrangian Relaxation. The solutions of these methods can be used either as approximations of the solutions returned by iterative algorithms such as subgradient descent and bundle method, or as *informed* starting points for such algorithms, saving many iterations. We evaluate our proposition on instances of the Multi-Commodity Fixed-Charge Network Design Problem and show the merits of our method.

Lagrangian Relaxation (LR) [1] is a well known method in optimization that allows to relax some hard constraints and thus reduce the complexity of the problem while providing, in many cases, a bound to the original problem better than the one computed by the continuous relaxation. Relaxed constraints are put into the objective function as soft penalizations whose coefficients are called the Lagrangian multipliers. The goal of LR is to find optimal coefficients, i.e. coefficients that provide the optimal bound. Unfortunately, most algorithms to solve LR are based on (sub-)gradient descent and are thus slow to converge in practice.

Machine Learning and Semi-Amortization In this presentation, we show that we can leverage machine learning techniques based on neural networks to predict Lagrangian Multipliers framed as Supervised and Unsupervised Learning. These predictions can be used in lieu of the Multipliers obtained by iterative algorithms. One of the key advantages is speed, where instead of dozens of steps of gradient descent a single forward pass over the network is required for each subproblem (which can well be parallelized on GPU). Unfortunately, learning accurate Multipliers is difficult and the predictions can be far from the optimal multipliers. Hence, we propose to use predictions as starting points for the exact iterative algorithms such as Volume or Bundle methods [2]. Hence our method appertains to the family of Semi-Amortized learning methods [3], that improve their prediction at inference time.

Evaluation We focus on the problem of Multi-Commodity Fixed-Charge Network Design Problem [4]. This problem consists on a multiflow problem over a graph in which each arc have a certain capacity, a fixed cost (that we pay for the activation of the arc) and a routing cost (that we pay to send an unitary amount of flow through this arc). It can be written as a Mixed-Integer Linear Programming Problem and here we use LR to relax flow constraints.

We implement different machine learning models for predicting the Lagrangian multipliers and compare their accuracy, and the improvement of the Bundle method when starting with the predicted multipliers.

References

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