

# Selecting transportation strategies in transportation planning for automotive supply chain.

Issa Bou Zeid<sup>1,2</sup>, Gülgün Alpan<sup>1</sup>, Bernard Penz<sup>1</sup>, Alain Nguyen<sup>2</sup>, Christian Serrano<sup>3</sup>

<sup>1</sup> Univ. Grenoble Alpes, CNRS, Grenoble INP <sup>†</sup>, G-SCOP, 38000 Grenoble, France

<sup>2</sup> Direction Transformation Digitale, Pôle optimisation, Renault, 92359 Le Plessis Robinson, France

<sup>3</sup> Direction Supply Chain, Renault, 78280 Guyancourt, France

{issa.bou-zeid,alain.nguyen,christian.serrano}@renault.com

{gulgun.alpan,bernard.penz}@grenoble-inp.fr

**Résumé:** Defining a transportation plan for inbound logistics in the automotive industry represents a major economic challenge for the manufacturers. A three phased algorithm, where in the second, three mixed integer linear programming models (MILP) are developed for the problem. To test the performances of the proposed approach, computational experiments are done on real-world instances, and the results are reported.

**Mots-clés:** *inbound logistics network, optimisation, vehicle routing problem*

## 1 Introduction

In the automotive industry, the Supply Chain (SC) is categorically divided into two major logistic processes, the inbound and the outbound SC. While, the inbound SC secures the effective supply of raw materials between suppliers and the plants, the outbound SC includes the delivery of final products to the clients. Recent increasing in global competition, has prompted manufacturers to be innovative in order to reduce costs, by optimizing the logistic processes. This article considers the problem of defining a least-cost inbound transportation plan subject to various operational and network constraints for a leading automobile manufacturer. The inbound logistics of the plants involves a significant number of suppliers as many of the parts needed to produce one vehicle are made by hundreds if not thousands of suppliers, forming the inbound logistics network. To supply the plants, a fleet of vehicles is used that is generally do not belong the automotive manufacturers due to, among others, cost reasons [1]. Thus the SC activities are outsourced to third party logistics (3PLs). A mixed inbound network with milk-run and indirect strategy transportation strategies is considered. A milk-run is a picked-up tour starts at one supplier, visits a subset of suppliers exactly once before ending at the plant. A special case of it, is the direct strategy, where the number of suppliers is equal to one. Finally, the indirect strategy delivers the plant through an intermediate facility, called cross-dock. Hence, in our problem, we aim on finding the transportation strategy for each supplier, and the decision variables related to optimal pick-up sequence, volumes, and frequencies.

This article is organized as follows. Section 2 provides a brief literature review. Section 3 describes in details the problem. Section 4 presents the solution approach. Section 5 presents the results of computational experiments, and finally, in section 6 conclusions are drawn, and future works are discussed.

## 2 Literature review

The studied problem is the transportation planning problem (TPP) with transportation strategies selection. The decision variables on transportation strategies selection is related to the class of Designing Transportation networks in the literature. [2] proposes an integrated mathematical model that addresses the assignment of frequencies, selection of transportation

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<sup>†</sup>Institute of Engineering Univ. Grenoble Alpes

strategies, and determination of milk-runs scheduling to mixed inbound networks with three strategies. Point-to-Point, Area Forwarding services, and milk runs are solved using a standard solver. [3] considers an inbound logistics planning problem with courier, express services and milk-runs transportation modes. A MILP is proposed together with valid inequalities to improve the convergence of the model towards optimal solutions. When suppliers are assigned to direct or milk-run strategies, decisions on pick-up sequence, frequency, and volumes have to be made for each vehicle route. These decisions are related to the vehicle routing problem (VRP), and its variants depending on the possible decisions that might be involved. The Flexible Periodic VRP allows service policies that are flexible with respect to the frequency of visits and the amount delivered at each visit [4]. The periodic VRP with service choice deals with flexibility in the minimum visit frequency [5]. The Inventory Routing Problem [6, 7], and the split-delivery VRP [8] allows flexible service policy on the amount picked-up at each visit. All these VRP classes find the optimal pickup sequence.

### 3 Problem description

The TTP is described as a set of destinations called receiving docks  $d \in \mathcal{D}$ , located in the same plant, which are supplied by a set of geographically dispersed sources  $s \in \mathcal{S}$  called suppliers. Demand requests  $r_{sd}$  exist from  $d$  to  $s$  expressed by a vector of three measuring units,  $l_{sd}$  in loading meter (LDM),  $w_{sd}$  in kilogram (Kg), and  $cm_{sd}$  in cubic meter ( $m^3$ ) that can be transported with a fleet of unlimited homogeneous vehicles  $\mathcal{V}$  having two loading capacity constraints  $Q$  for LDM and  $KG$  for Kg.

To transport  $r_{sd}$ , either indirect strategy or milk-run strategy is used. For the indirect strategy, suppliers are assigned to their respective cross-docks. For the milk-run strategy, the fleet  $\mathcal{V}$  is used to transport the demands from suppliers to their respective receiving docks. Each transportation strategy has its own cost.

On one hand, indirect cost  $\delta_s$  for supplier  $s$ , is composed of the upstream cost between supplier  $s$  and its respective cross-dock that depends on  $\sum_d cm_{sd}$  and the distance  $s$  to its cross-dock, the handling cost of  $\sum_d cm_{sd}$  for supplier  $s$  inside the cross-dock calculated for every  $1 m^3$ , and the downstream cost between the cross-dock and the plant calculated by first adding the fixed cost of using one vehicle to the cost of traveling between cross-dock and plant, then multiplied by the number of vehicles needed. Given  $r_{sd}$ ,  $\delta_s$  is precalculated. On the other hand, milk-run cost is calculated for every used  $v \in \mathcal{V}$ . It is the sum of the variable cost of the route between a set of suppliers and the plant, the fixed cost of using one vehicle, and the fixed cost of the number of stops (suppliers).

We define the TTP on an undirected graph  $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ , where  $\mathcal{N}$  is the vertex set, and  $\mathcal{A} = \{(s_i, s_j) : s_i, s_j \in \mathcal{N}, s_i \neq s_j\}$  is the arcs set that connects the nodes in the graph. Let  $\mathcal{N} = \mathcal{S} \cup \{0\}$ , with  $\mathcal{S} = \{s_1, \dots, s_{|\mathcal{S}|}\}$ , and  $\{0\}$  is the plant where docks in  $\mathcal{D}$  are located. Every node  $s_i \in \mathcal{S}$ , is associated with a pick-up vector of  $r_{sd}$  to every  $d \in \mathcal{D}$ . Three kinds of arcs exists, each associated with a cost. Arcs  $(0, s_i)$  between the plant and suppliers have a cost  $c_{0,s_i} = 0$  (because the pick-up tour starts at the suppliers), arcs  $(s_i, s_j)$  between the suppliers have a cost  $c_{s_i,s_j} \geq 0$ , which is the travel cost from  $s_i$  to  $s_j$ , and arcs  $(s_i, 0)$  connecting the suppliers and the plant have a cost of  $c_{s_i,0} \geq 0$ , the travel cost from  $s_i$  to 0. For milk-run suppliers, the optimal elementary routes that minimize the total costs are obtained for every  $v \in \mathcal{V}$ . We find the elementary routes, by assuring that it starts and ends at the plant  $\{0\}$ , and visits a set of suppliers exactly once with no cycles. Fig. 1a illustrates an example of a small graph  $\mathcal{G}$ , with  $\mathcal{N} = 4$ , and  $\mathcal{D} = 2$ , and Fig. 1b shows an example of a feasible solution for vehicle  $v$  in  $\mathcal{G}$ .

Our objective is therefore to define a transportation plan that satisfies the constraints and minimizes total milk-run and indirect costs. First, it is assumed that if an indirect strategy is chosen for a supplier, an explicit route need not be found, and the precomputed cost  $\delta_s$  is applied. For each supplier  $s$  and receiving dock  $d$ , with  $r_{sd} > 0$ , a minimum visit frequency  $f_{sd}$  is imposed in order to have a higher number of small deliveries, and consequently a low inventory at the plant [9]. In addition, for practical industrial reasons, suppliers are divided

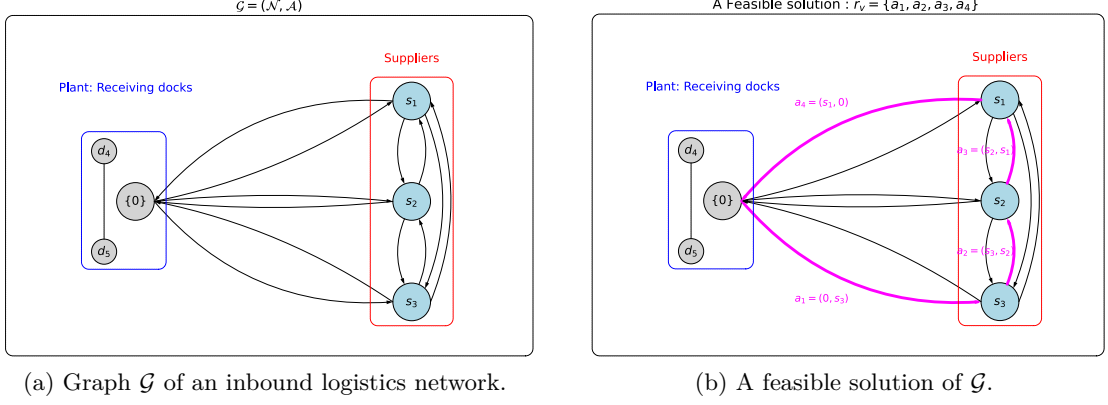


FIG. 1 – An example of a small graph  $\mathcal{G}$ , with a feasible solution.

into clusters of maximum size  $MaxC$ , and one supplier belongs to exactly one cluster. Finally, for each vehicle  $v$  and its associated route,  $MaxS$  is the maximum number of stops (suppliers in the route),  $MaxD$  is the maximum number of docks supplied, and the distance between two consecutive stops cannot exceed  $MaxQ$ .

## 4 Solution Approach

Due to the complexity of the problem, we propose a three-phase approach. In phase 1, the clustering phase, we generate all possible clusters from the set of suppliers  $\mathcal{S}$  ( $c \in \mathcal{C}$ ), respecting the distance constraints. For each cluster  $c \in \mathcal{C}$ , the subgraph  $\mathcal{G}'_c$  is constructed. In phase 2, the optimal transportation plan for each cluster is found by a MILP. In phase 3, a Set Partitioning Problem is solved to find the best set of clusters that minimizes the total cost.

**4.1 Phase 1 : Clustering** For each subset  $\mathcal{S}' \subseteq \mathcal{S}$  such that  $1 \leq |\mathcal{S}'| \leq MaxC$ , the corresponding cluster  $c$  is validated ( $c \in \mathcal{C}$ ) if the associated subgraph  $\mathcal{G}'_c$  is connected.

**4.2 Phase 2 : Routing and Transportation Strategies** The optimal transportation plan cost for each cluster  $c \in \mathcal{C}$  is obtained by determining the transportation strategy for the suppliers, by solving independently a MILP on each subgraph  $\mathcal{G}'_c$ .

Before introducing the MILP, parameters and decision variables are presented. For the parameters,  $c_{ij}$  ( $(i, j) \in \mathcal{N}$ ) is the cost of the arc  $(i, j)$ ,  $\delta_s$  ( $s \in \mathcal{S}$ ) is the cost of assigning supplier  $s$  to indirect strategy,  $S$  and  $F$  are the fixed cost per stop at a supplier and the fixed cost per vehicle respectively. A big  $M$  quantity is mandatory in the MILP for each supplier  $s \in \mathcal{S}$  :  $\bar{U}_s = \min(Q, \sum_d l_{sd})$ .

Decision variables are as follows :

- $x_{ij}^v$  : 1 if the arc  $(i, j) \in \mathcal{N}$  is visited by vehicle  $v \in \mathcal{V}$ , 0 otherwise ;
- $y_s^v$  : 1 if supplier  $s \in \mathcal{S}$  is visited by vehicle  $v \in \mathcal{V}$ , 0 otherwise ;
- $z_{sd}^v$  : Quantity supplied from supplier  $s \in \mathcal{S}$  to dock  $d \in \mathcal{D}$  by vehicle  $v \in \mathcal{V}$  ( $z_{sd}^v \in \mathbb{R}^+$ ) ;
- $t_{sd}^v$  : 1 if supplier  $s \in \mathcal{S}$  and receiving dock  $d$  are visited by vehicle  $v \in \mathcal{V}$ , 0 otherwise ;
- $p_d^v$  : 1 if dock  $d \in \mathcal{D}$  is visited by vehicle  $v \in \mathcal{V}$ , 0 otherwise ;
- $u^v$  : 1 if vehicle  $v \in \mathcal{V}$  is used, 0 otherwise ;
- $s^v$  : Number of stops for vehicle  $v \in \mathcal{V}$  ( $s^v \in \mathbb{N}^+$ ) ;
- $\gamma_s$  : 1 if supplier  $s \in \mathcal{S}$  is assigned to milk-run strategy, 0 for the indirect strategy.

$$\text{Minimize} \quad \sum_{i,j \in \mathcal{N}} \sum_{v \in \mathcal{V}} x_{ij}^v c_{ij} + S \sum_{v \in \mathcal{V}} s^v + F \sum_{v \in \mathcal{V}} u^v + \sum_{s \in \mathcal{S}} \delta_s (1 - \gamma_s) \quad (1)$$

$$\gamma_s \leq \sum_v y_s^v \quad \forall s \in \mathcal{S} \quad (2)$$

$$\sum_s \sum_d z_{sd}^v \leq Q \quad \forall v \in \mathcal{V} \quad (3)$$

$$\sum_s \sum_d \frac{w_{sd}}{d_{sd}} z_{sd}^v \leq KG \quad \forall v \in \mathcal{V} \quad (4)$$

$$\sum_d z_{sd}^v \leq \bar{U}_s y_s^v \quad \forall s \in \mathcal{S}, v \in \mathcal{V} \quad (5)$$

$$\sum_v z_{sd}^v = l_{sd} \gamma_s \quad \forall s \in \mathcal{S}, d \in \mathcal{D} \quad (6)$$

$$\sum_s y_s^v \leq MaxS u^v \quad \forall v \in \mathcal{V} \quad (7)$$

$$u^v \leq \sum_s y_s^v \quad \forall v \in \mathcal{V} \quad (8)$$

$$M p_d^v \geq \sum_s t_{sd}^v \quad \forall d \in \mathcal{D}, v \in \mathcal{V} \quad (9)$$

$$p_d^v \leq \sum_s t_{sd}^v \quad \forall d \in \mathcal{D}, v \in \mathcal{V} \quad (10)$$

$$\sum_d p_d^v \leq MaxD u^v \quad \forall v \in \mathcal{V} \quad (11)$$

$$z_{sd}^v \leq Q t_{sd}^v \quad \forall s \in \mathcal{S}, d \in \mathcal{D}, v \in \mathcal{V} \quad (12)$$

$$z_{sd}^v \geq t_{sd}^v \quad \forall s \in \mathcal{S}, d \in \mathcal{D}, v \in \mathcal{V} \quad (13)$$

$$\sum_v t_{sd}^v \geq f_{sd} \gamma_s \quad \forall s \in \mathcal{S}, d \in \mathcal{D} \quad (14)$$

$$y_0^v \geq u^v \quad \forall v \in \mathcal{V} \quad (15)$$

$$\sum_{j \in \mathcal{N}} x_{ij}^v = y_i^v \quad \forall i \in \mathcal{N}, i \neq j, v \in \mathcal{V} \quad (16)$$

$$\sum_{i \in \mathcal{N}} x_{ij}^v = y_j^v \quad \forall j \in \mathcal{N}, i \neq j, v \in \mathcal{V} \quad (17)$$

$$\sum_{i,j \in \mathcal{I}} x_{ij}^v \leq |\mathcal{I}| - 1 \quad \forall \mathcal{I} \subseteq \mathcal{S}, i \neq j, v \in \mathcal{V} \quad (18)$$

$$s^v \geq \sum_{s \in \mathcal{S}} y_s^v - 1 \quad \forall v \in \mathcal{V} \quad (19)$$

The objective function (1) minimizes the total milk-run and indirect costs for the given cluster. Constraint (2) guarantees that if supplier  $s$  is not visited in any vehicle  $v$ , then it is assigned to indirect strategy. Constraints (3,4) guarantees that the vehicle capacities in LDM and Kg respectively are respected. Constraint (5) ensures that if supplier  $s$  is visited by vehicle  $v$ , at most  $\bar{U}_s$  can be collected. Constraint (6) satisfies the total demand request for the pair  $(s, d)$ . Constraint(7,8) respects the maximum number of suppliers in vehicle  $v$ , and forces  $u^v$  to be 1 if at least one supplier is visited. Constraints (9,10) determines when a receiving dock  $d$  is visited by vehicle  $v$ . Constraint (11) respects the maximum number of receiving dock per vehicle  $v$ . Constraints (12,13) ensures that if a positive amount of demand for dock  $d$  is collected from supplier  $s$  by vehicle  $v$ , then vehicle  $v$  contains both supplier  $s$  and receiving dock  $d$ . Constraint (14) respects the minimum frequency for the pair  $(s, d)$ . Constraint (15) guarantees that the plant is visited only if vehicle  $v$  is used. Constraints (16,17) guarantee that for the plant, and for each supplier there's one sorting, and one entering arc. Constraint (18) eliminates the sub-tours. Constraint (19) finds the number of stops in vehicle  $v$ .

**4.3 Phase 3 : Set Partitioning Problem (SPP)** The Set Partitioning Problem (SPP), determines the best subset of clusters  $\mathcal{C}' \subseteq \mathcal{C}$ , such that each supplier  $s \in \mathcal{S}$  belongs exactly to one cluster  $c \in \mathcal{C}'$ , and the total cost is minimum. Let  $\alpha_c$  be the cost of every  $c \in \mathcal{C}$ , parameter  $a_{sc} = 1$  if supplier  $s$  is in cluster  $c$ , and variable  $x_c = 1$  if cluster  $c$  is selected in the optimal solution, 0 otherwise. The integer linear programming model (ILP) for the [SPP] is :

$$[\text{SPP}] \text{ Minimize } \sum_{c \in \mathcal{C}} \alpha_c x_c \quad (20)$$

$$\sum_{c \in \mathcal{C}} a_{sc} x_c = 1 \quad \forall s \in \mathcal{S} \quad (21)$$

**4.4 Enhanced formulation** The above model can be further strengthened by using reformulation techniques for some constraints and introducing new ones. The first technique used is disaggregation. The basic idea is to separate one constraint (big-M type constraints) into many without changing the meaning of a MIP model. We use this technique to reformulate constraints 2, 7, and 9. The sub-tour elimination in 18 is also reformulated. It prevents sub-tours by enumerating all subsets  $\mathcal{I}$  of size  $2 \leq |\mathcal{I}| \leq |\mathcal{N}| - 1$ , and ensures the number of arcs selected is less than the number of nodes - 1. However, due to the *MaxS* constraint, it is sufficient to only eliminate subsets  $\mathcal{I} : 2 \leq |\mathcal{I}| \leq \text{MaxS}$ . It can be done by finding cliques and cycles of size  $k : 2 \leq k \leq \text{MaxS}$ . In addition, the right-hand side of the constraint is replaced by summing on  $y_s^v$  where  $s \in \phi$ , with  $\phi$  is the subset of all combination of suppliers from  $\mathcal{I}$  with  $|\phi| = |\mathcal{I}| - 1$  providing a tighter formulation. The new constraint is :  $\sum_{(i,j) \in \mathcal{I}} x_{ij}^v \leq \sum_{s \in \phi} y_s^v, \forall \mathcal{I} \subseteq \mathcal{S}, 2 \leq |\mathcal{I}| \leq \text{MaxS}, \phi \subseteq \mathcal{I}, |\phi| = |\mathcal{I}| - 1, v \in \mathcal{V}$ . We also introduce a valid inequality. Let  $\psi$  be the set of all shortest paths for every pair  $(s, s'), s, s' \in \mathcal{S}$  with a number of nodes greater than  $\text{MaxS} - 1$ . Then :  $y_s^v + y_{s'}^v \leq 1, \forall s, s' \in \psi, s \neq s', v \in \mathcal{V}$  is valid. In other words, if for every pair  $(s, s')$ , supplier  $s$  cannot reach supplier  $s'$  in less than  $\text{MaxS} - 1$  stops, then  $s$  and  $s'$  cannot be in the same vehicle  $v$ . A symmetry break constraint is also introduced. It imposes that vehicle  $v + 1$  cannot be used if vehicle  $v$  is not yet used,  $u^v \geq u^{v+1}, \forall v \in \mathcal{V}$ .

## 5 Computational Experiments and Results

To evaluate the performance of the formulations, computational experiments are done on real-world instances conducted on a personal PC equipped with an 11<sup>th</sup> generation Intel Core i7 3.00 GHz , 8 logical processor, and 32GB of random-access memory (RAM). The MILP and ILP models are solved using the commercial solver CPLEX, with a maximum time limit for every cluster (TLC) in phase 2, of 180 seconds, and a relative tolerance on the gap between the best integer objective and the objective of the best node remaining of 0.5% for small and medium-sized instances, and 1% to large-sized instances. In addition, a maximum time limit for every instance (TLI) of 7200 seconds (2 hours) for small and medium-sized instances, and 25200 seconds (7 hours) for large-sized instances is also set. Besides, phase 2 of the algorithm is parallelized on two processors using the Fork/Join framework in Java.

The real-world instances are divided into small, medium and large-sized instances, distinctive in the size of  $\mathcal{S}$ ,  $\mathcal{D}$ ,  $\mathcal{N}$  (graph density),  $\mathcal{C}$ ,  $r_{sd}$  (number of  $\mathcal{V}$ ). For example, the size of  $\mathcal{S}$  varies between 10 and 54, and the minimum, maximum and number of  $V$  for clusters are 1 and 85 respectively. The average number of vehicles per cluster varies between, 3.33 and 14.4.

Table 1 shows the results of the enhanced model compared to the original one. The minimum, maximum and average improvement are 43.16%, 95.91% and 75.06%. In addition, the enhanced model has improved the bound of several clusters, solving them optimally. For example, for instance number 4, 9 clusters that exceeded the TLC are solved to optimality with the enhanced model. Finally, the number of unsolved clusters has also been improved. the minimum, maximum and average improvement percentage is, 9.3%, 100% and 23.7% respectively. In summary, although the enhanced model has improved the computational time of 57% of the instances, it has limitation on medium and large-sized instances.

## 6 Conclusions and perspectives

A three-phase approach is proposed for an inbound SC problem, giving good results small industrial instances. However, solving medium and large-sized instances is challenging because of the : 1) significant number of clusters, 2) clusters with high volumes are more difficult to be solved. The weakness of the approach, comes from solving the MILP that jointly finds for every

TAB. 1 – Computational comparison between the two models.

Type	I <sup>1</sup>	S <sup>2</sup>	D <sup>3</sup>	N <sup>4</sup>	C <sup>5</sup>	Original model			Enhanced model			$\Delta_T$ <sup>9</sup>	$\Delta_\phi$ <sup>10</sup>	$\Delta_S$ <sup>11</sup>
						T <sup>6</sup>	$\emptyset$ <sup>7</sup>	S <sup>8</sup>	T	$\emptyset$	S			
<b>Small</b>	1	10	4	37	293	28.71	0	0	15.05	0	0	47.58	0	0
	2	12	3	51	688	3541.20	2	0	144.66	0	0	95.91	100	0
	3	13	2	55	846	89.78	0	0	28.74	0	0	67.99	0	0
	4	14	5	91	1470	7651.19	9	0	649.31	0	0	91.51	100	0
	5	14	4	46	285	3993.76	3	0	215.37	0	0	94.61	100	0
	6	15	7	86	1776	2045.46	1	0	260.96	0	0	87.24	100	0
	7	15	5	104	1939	519.66	0	0	130.47	0	0	74.89	0	0
<b>Medium</b>	8	19	4	171	5035	TLI <sup>12</sup>	8	3505	TLI	0	831	0	100	76.3
	9	25	8	276	9617	TLI	11	2225	TLI	11	823	0	0	63.0
	10	33	10	217	35243	TLI	4	15587	TLI	3	7646	0	25	50.9
	11	34	5	520	51887	TLI	12	31808	TLI	12	27192	0	0	14.5
	12	34	6	553	52932	TLI	3	29726	4092.17	0	0	43.16	100	100
<b>Large</b>	13	50	9	687	118788	TLI	12	76145	TLI	11	69041	0	8.33	9.3
	14	54	10	721	128147	TLI	6	85793	TLI	1	70222	0	83.33	18.1

<sup>1</sup> Instance number, <sup>2</sup> Num. suppliers, <sup>3</sup> Num. receiving docks, <sup>4</sup> Num. edges, <sup>5</sup> Num. of clusters, <sup>6</sup> Computational time in seconds, <sup>7</sup> Num. of clusters exceeded TLC, <sup>8</sup> Num. of clusters unsolved within TLI, <sup>9</sup> Percentage decrease in computational time = (Worst time - Best time) / Worst time  $\times$  100, <sup>10</sup> Percentage decrease in the number of not optimal clusters, <sup>11</sup> Percentage decrease in the number of unsolved clusters, <sup>12</sup> Instances exceeded TLI.

supplier the optimal transportation strategy, and for every vehicle the shortest elementary paths, and its optimal loading plan. Hence, another approach that first directly generates interesting routes, and then choose the optimal ones has to be considered.

## Références

- [1] P.P. Repoussis, C.D. Tarantilis, O. Bräysy, and G. Ioannou. A hybrid evolution strategy for the open vehicle routing problem. *Computers & Operations Research*, 37(3) :443–455, 2010.
- [2] Anne Meyer and Boris Amberg. Transport concept selection considering supplier milk runs—an integrated model and a case study from the automotive industry. *Transportation Research Part E : Logistics and Transportation Review*, 113 :147–169, 2018.
- [3] Reinhard Baller, Pirmin Fontaine, Stefan Minner, and Zhen Lai. Optimizing automotive inbound logistics : A mixed-integer linear programming approach. *Transportation Research Part E : Logistics and Transportation Review*, 163 :102734, 2022.
- [4] Diana Lucia Huerta Muñoz. *The flexible periodic vehicle routing problem : modeling alternatives and solution techniques*. PhD thesis, Universitat Politècnica de Catalunya, 2018.
- [5] Peter Francis, Karen Smilowitz, and Michal Tzur. The period vehicle routing problem with service choice. *Transportation Science*, 40(4) :439–454, 2006.
- [6] Luca Bertazzi and M. Grazia Speranza. Inventory routing problems with multiple customers. *EURO Journal on Transportation and Logistics*, 2(3) :255–275, 2013.
- [7] Leandro C. Coelho, Jean-François Cordeau, and Gilbert Laporte. Thirty years of inventory routing. *Transportation Science*, 48(1) :1–19, 2014.
- [8] Moshe Dror and Pierre Trudeau. Savings by split delivery routing. *Transportation Science*, 23(2) :141–145, 1989.
- [9] Ford W. Harris. How many parts to make at once. *Operations Research*, 38(6) :947–950, 1990.