# Modeling, performance evaluation and decision support of disassembly processes using BDSPNs 

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## 1 Introduction

Recent research has trained a number of disassembly problems, including the disassembly sequencing problem [1], the disassembly line balancing problem [2], the disassembly scheduling problems [3], the disassembly leveling problems [4], and the disassembly lot-sizing problem [5]. Among the various problems related to disassembly systems, our paper focuses on the disassembly lotsizing problem which can be defined as "the problem of determining the quantity and timing of End-Of-Life (EOL) products to be disassembled in order to satisfy the demand of their parts or components over a given planning horizon" [6]. Therefore, to optimize the performance indicators, it is necessary to decide when and how many EOL products should be disassembled to meet the demand for the components [5]. In this research, we study a multi-product disassembly lot-sizing problem under stochastic batch demand using a Batch Deterministic and Stochastic Petri Net (BDSPN). The two-level product structure is considered. The main goal is to determine the quantity of disassembling products so as to satisfy the time-varying demands of components / parts under stochastic and batch demands. i.e., which batch customer's order should be disassembled first for different products. In our study, an analytical approach is adopted to analytically evaluate the performance of our system. BDSPNs are then applied for modeling and for the performance evaluation of the disassembly process. The main contributions related to this work are:

- A new class of high-level Petri networks, called the BDSPN is adopted to model the lot-sizing disassembly problem under a stochastic process.
- The change of resources (material or human) in the different flows characterizing the processes of disassembly process is taken into consideration in our proposed algorithm.
- A new mathematical model is put forward which aims to minimize the sum of extra costs due to the change of resources (human or material), disassembly operations, and holding costs for multi types of products.
- The potential synergies between PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) and BDSPNs are investigated in order to aid decision-makers to classify optimal disassembly scenarios based on performance indicators. This is the first time that PROMETHEE and BDSPNs are used simultaneously.
- A real case study of the manufacturing company CODIMATRA is studied to improve the efficacity of the proposed approach


## 2 BDSPNS FORMALISMS AND PERFORMANCE ANALYSIS

BDSPN [7] is a nine tuple and it is noted as: $N=\left(P, T, I, O, V, W, I I, D, u_{0}\right)$. There are two types of places: discrete places in $p_{d}$ and batch places in $p_{b}$. Each batch token is represented by a number that indicates its size. However, tokens in discrete places are the same as those in standard PNs. There are also two types of transitions, $T_{i}$ presents an immediate transition, and $T_{d}$ denotes the deterministically timed transitions, and $T_{e}$ represents the stochastic transitions with an exponentially distributed firing time. In BDSPN, $\mathrm{I} \subseteq(\mathrm{P} \times \mathrm{T})$ define the input arcs and $\mathrm{O} \subseteq(\mathrm{T} \times \mathrm{P})$ is the output arcs. $V \subseteq(\mathrm{P} \times \mathrm{T})$ presents the inhibitor arcs of the transitions and $W$ defines the weights for ordinary arcs and inhibitor arcs. $\mathrm{I} \subseteq \Pi: \mathrm{T} \rightarrow \mathrm{Z}$ is a priority function assigning a priority to each transition. Timed transitions are assumed to have the lowest priority. D: $\mathrm{T} \rightarrow\{0\} \mathrm{U} I R+\mathrm{U} \operatorname{Exp}$ defines the firing times of all transitions. $\mu_{0}$ is the initial $\mu$-marking of the net, where $2^{\text {IN }}$ consists of all subsets of $Z$.
The evaluation of this process is based on the temporal evolution of the model $\mu$-marking process [8]. An analytical approach based on the graph of $\mu$-markings. This graph is used in conjunction with stochastic processes to get benefit from their evaluation methods, including Markov processes. This procedure is particularly applicable when there are a finite number of states ( $\mu$-marking). The general procedure is described by the following steps [8].
1- Construct the graph of $\mu$-markings. Batch transitions are marked by their crossing indices ( $t_{j}[\mathrm{q}]$ ).
2- Eliminate the unstable states (nodes), and the associated immediate transitions (arcs).
3- Obtain the stochastic marking process noted by $\mu$ (t) from the reduced $\mu$-reachability graph. The
4- Determine the steady-state distribution of the stochastic process.
5- Evaluate the performance indexes that characterize the BDSPN modeled process by knowing the stationary probability distribution of their states (obtained in Step 4).

## 3 A new approach for modeling a disassembly lot-sizing problem

### 3.1 Problem statement

This paper aims to determine which batch demand should be first disassembled for each type of product to ensure the quantity of disassembling root items so as to satisfy the time-varying batch demands of leaf items over the planning horizon. Some performance indicators based on a reference static model are calculated. Then, this paper proposes a new mathematical model that aims to minimize the sum of extra cost due to the change of resources (human or material), disassembly operation, and holding costs for multi types of products, over all scenarios. A Multi-Criteria Decision Making (MCDM) technique is proposed to the disassembly process. The PROMETHEE II method is used for prioritizing the suggested scenarios. Two criteria have been considered for this problem: the cost and the frequency of each scenario.

### 3.2 Mathematical formalization

The notations are summarized as follows:
$i, q \quad$ Index for items $l$ (the final components) $\in(1,2, \ldots, N)$ and Batch firing index respectively
$K$ : $\quad$ Index of operation $k=\{1, \ldots, K\}$
$D T_{b j l}$ : The disassembly operation cost of leaf item $l$ of product $j$ for batch firing index $b$
$h_{q j k}$ : Extra cost to execute operation k of product $j$ for batch firing index q due to the change of resource
$H_{j l b}$ : Holding cost of leaf items disassembled from root item l of product j for batch firing index b
$d_{l j}, \Phi:$ Demand of leaf item $l$ of product $j$ and different sizes of batch orders
$a_{l j}$ : Number of units of item l obtained by disassembly of one unit of root item $j$
$q_{i j}: \quad$ Transition rate from state $m_{i}$ to state $m_{j}$ of incidence matrix $Q$
$I_{l t}$ : Inventory level of leaf item $l$ at the end of transition $t$

## Decision variables

$Y_{q j}: \quad Y_{q j}=1$, if performing operation $k$ after its previous operation $k-1$, otherwise, $Y_{q j}=0$;
$Z_{b j l}$ : Quantity of leaf item disassembled from root item $l$ of product $j$ for batch firing index $b$ to satisfy demand;
$F_{l t}$ : Disposed quantity of leaf item 1 in period $t$;
$E_{b j k}: 1$ if there is an extra cost to execute operation k of product j for batch firing index b , and 0 otherwise;

The formulation of the problem is the following:

$$
\begin{gathered}
{[P 1] \operatorname{Min}\left\{\left(\sum_{j=1}^{J} \sum_{i=1}^{N} \sum_{b \epsilon \Phi} H_{j q i} \quad W(P, t) N L(b, P)_{\operatorname{moy} i}+\right.\right.} \\
\sum_{j=1}^{J} \sum_{i=1}^{N} \sum_{b \epsilon \Phi} \sum_{B D S P N s} D T_{j i} b \operatorname{Card}\left(\mu_{i}(p)\right) \pi_{i}+\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{b \epsilon \Phi} b \quad h_{b j k} E_{b j k} \pi_{m}(6)
\end{gathered}
$$

Subject to:

$$
\begin{array}{lr}
I_{l t}=I_{l t-1}+\sum_{j=1}^{J} a_{l j} q . b-F_{l t}-d_{l j} & \forall l=1 \ldots N \& \forall t=1,2 \ldots T \\
\sum_{b \in \Phi} Z_{j q l}=d_{l j} & \forall l=1 \ldots \mathrm{~N} \quad \& \forall j=1,2 \ldots J \\
a_{l j} \cdot q \cdot b \geq \sum_{b \in \Phi} Z_{j q l} & \forall l=1 \ldots N \& \quad \forall j=1,2 \ldots J \\
w_{b j k}=0 \text { or } 1 & \forall j=1,2 \ldots J \quad \& k=1 \ldots K \\
Z_{j b l} \geq 0 & \forall l=1 \ldots N \& \quad k \in \Phi(i) \& \quad 1=1 \ldots N \tag{11}
\end{array}
$$

The objective function (6) aims to minimize the sum of extra costs, disassembly operations, and holding costs for multi types of products. Constraints (7) define the inventory balance equations for the components. Constraints (8) show that the demands of leaf items should be satisfied. Constraints (9) express that the total quantity of leaf item 1 obtained by root item j , after product disassembly, will satisfy the demand. Constraints (10-11) represent the domains of decision variables.

## 4 Case study

### 4.1 BDSPNs of disassembly process

To test the effectiveness of the investigated approach, the case study of the mechanical company CODIMATRA [10] is chosen as an actual case. The studied company attends the return of defective machines from customers. After their collection, storage, and evaluation, an expert decides whether to sell them to a secondary consumer or to disassemble them. In the latter case, the machines are disassembled into pieces. A major part of the reverse logistics process is covered by this company. However, in our work, we will focus only on the stages of the disassembly process especially product storage and component disassembly and storage. The case of two types of products A and B, with twolevel product will be treated. Each product is composed of 4 components $\{\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4\}$. The disassembly cost of products A and B is respectively $33 €$ et $35 €$ and for each component varies between $5 €$ and $11 €$. The inventory cost for final components is constant, which is $20 €$ for all types of components. These costs vary depending on the batch number. However, extra costs due to the change
of resources to perform each batch demand varies according to customer orders, disassembly operations, and component storage. These extra costs vary from $6 €$ to $19 €$

|  | Product A |  |  |  | Product B |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P_{4}$ | $P_{5}$ | $P_{6}$ | $P_{7}$ | $P_{8}$ | $P_{9}$ | $P_{10}$ | $P_{11}$ |
| $\mathrm{~b}=40,70$ | 4 | 2 | 3 | 2 | 1 | 2 | 2 | 3 |

Table 1: Nomenclature coefficient

### 4.2 BDSPNs of disassembly process

The batch disassembly process of two types of products in this study is shown in figure 1 and has the following characteristics. The arc inhibitor combined to place $p_{1}$ and $p_{3}$ whose weight corresponds to the threshold of disassembly operation. Thus, it controls the inventory level, $\mathrm{M}\left(p_{1}\right)$ and $\mathrm{M}\left(p_{2}\right)$. Transitions $t_{3}, t_{4}$ presented the disassembly orders. Discrete places $p_{1}, p_{3}$ used to represent the stock of component A and B , respectively; A batch place $p_{2}$ used to represent customer orders with different sizes. The order size to be processed for the two types of products may be the same or different. It follows a random choice policy. That is to say, each batch token validating the transition can cross the exit transition with a given probability with the restriction that if two batch tokens have the same size, only one of the two will be able to cross the arc. In our example, we have two batch order sizes, 40 and 70. The batch deposit of the finished component are modeled by places $p_{4} \ldots p_{11}$ and $t_{5} \ldots t_{12}$ represent the end of batch disassembly operation and the start of the storage of the batch. The storage of ready final components, which were the disassembled products A and B are modeled by the batch places $p_{12} \ldots p_{19}$. Then $t_{13}$ and $t_{14}$ give the orders to start a new batch order for products A and B .

### 4.3 Evolution and resolution of associated stochastic process

The evolution of the process can be expressed by using the graph of the $\mu$-marking BDSPN model that represents this process. Each $\mu$ marking of the graph represents the state of the process, and each crossing between two $\mu$-markings in the graph represents the execution of an operation that can be the start of the disassembly operation for A (crossing $t_{2}$ ), the end of the disassembly operation or the start of storing the batch components (crossing $t_{5}$ ). Each batch transition is marked by its corresponding batch firing index $q$. This graph is built from the initial $\mu$-marking $\mu 0$ considering all possible crossings of one $\mu$-marking to another. The immediate transitions are more prioritized than timed transitions. The resulting graph contains 21 tangible states numbered from 0 to 20. After that, by converting the reduced $\mu$-reachability graph to its corresponding stochastic process, we get a Continuous Timed Markov Chain (CTMC) that is depicted in figure 2. For this process, we will be limited to the following case:
$-\lambda 3_{[40]}=\lambda 1=0.5 ; \lambda 3_{[70]}=\lambda 2=0.7 ; \lambda 4_{[40]}=\lambda 3=0.8$ and $\lambda 4_{[70]}=\lambda 4=0.4$ means that the arrival of different types of demand orders of the two product follows an exponential law of different parameters.
$-\lambda 5_{[160]}=\lambda 5_{[280]}=\lambda 6_{[80]}=\lambda 6_{[140]}=\lambda 7_{[120]}=\lambda 7_{[210]}=\lambda 8_{[140]}=\lambda 8_{[80]}=\lambda 5=1$ and $\lambda 9_{[80]}=$ $\lambda 9_{[140]}=\lambda 10_{[120]}=\lambda 10_{[210]}=\lambda 11_{[80]}=\lambda 11_{[140]}=\lambda 12_{[80]}=\lambda 12_{[140]}=\lambda 6=0.5$ means that the storage operation depends on the number of tokens and the type of product.
$-\lambda 13_{[70]}=\lambda 13_{[40]}=\lambda 14_{[70]}=\lambda 14_{[40]}=\lambda 7=2$ means that the order to start a new batch of the two products follows an exponential law of identical parameters.
We calculate the infinitesimal generator matrix associated with the stochastic $\mu$-marking process : $\pi . \mathrm{Q}=0$ and $\sum_{i=0}^{20} \pi_{i}=1$, where $\pi=\left[\pi_{0}, \ldots, \pi_{i}, \ldots, \pi_{20}\right]$ is a row vector corresponding to the probability distribution of the states of the stochastic process of the $\mu$-marking, and Q is the transition matrix
associated with the Markov chain. The probability distribution of states $\sum_{i=0}^{20} \pi_{i}=\left[\begin{array}{ll}0.3125 & 0.0930\end{array}\right.$ $\begin{array}{llllllllllll}0.0546 & 0.125 & 0.0625 & 0.0194 & 0.0194 & 0.0194 & 0.0194 & 0.0194 & 0.0194 & 0.0194 & 0.0195 & 0.0311\end{array}$ $\begin{array}{lllllll}0.0311 & 0.0311 & 0.0311 & 0.0311 & 0.0311 & 0.0311 & 0.0311] .\end{array}$


Figure 1: BDSPN model of the proposed process


Figure 2: Associated markov chain

The arc inhibitor combined to place $p_{1}$ and $p_{3}$ whose weight corresponds to the threshold of disassembly operation. Thus, it controls the inventory level, $\mathrm{M}\left(p_{1}\right)$ and $\mathrm{M}\left(p_{2}\right)$. Transitions $t_{3}, t_{4}$ presented the disassembly orders. Discrete places $p_{1}, p_{3}$ used to represent the stock of component $A$ and $B$, respectively. A batch place $p_{2}$ used to represent customer orders with different sizes. The order size to be processed for the two types of products may be the same or different. It follows a random choice policy. That is to say, each batch token validating the transition can cross the exit transition with a given probability with the restriction that if two batch tokens have the same size, only one of the two will be able to cross the arc. In our example, we have two batch order sizes, 40 and 70 . The batch deposit of the finished component are modeled by places $p_{4}, \ldots, p_{11}$ and $t_{5} \ldots t_{12}$ represent the end of batch disassembly operation and the start of the storage of the batch. The storage of ready final components, which were the disassembled products A and B are modeled by the batch places $p_{12}, \ldots$, $p_{19}$. Then $t_{13}$ and $t_{14}$ give the orders to start a new batch order for products A and B .

### 4.4 Selection of optimal scenario using PROMETHEE II

The batch customer orders are subject to random requests. According to our example and the BDSPN marking graph, we have four scenarios. The PROMETHEE II over-classification method is chosen to complete the comparative study of the different possible scenarios The results for scenario 1 and 2 are respectively 8.099 euro and 7.7867 euro. Scenario 3 equal to 7.3294 euro and scenario 4 equal to 7.6878 euro. The most optimal scenario is scenario 3 which has a positive flow ( $\varphi^{+}=1,000$ ), while scenario 4 is located second with a positive flow ( $\varphi^{+}=0.333$ ). The third and the fourth ranks are for scenario $2\left(\varphi^{-}=0,333\right)$ and scenario $1\left(\varphi^{-}=-1,000\right)$, respectively. Judging the obtained results, it is noted that scenario 4 is the best-ranked alternative. This means that starting by launching a batch order of size 70 for product A and size 40 for product $B$ is the most favourable scenario. On the other hand, according to the experts' estimates, the worst-ranked alternative is alternative 1 (scenario 1 ).

## 5 Conclusion

This paper proposes a new approach to solve a disassembly lot-sizing problem under stochastic batch demands for multi products with a two-level structure. The BDSPN is used as a modelling tool. It
allows describing disassembly activities such as customer order processing, replenishment of stocks, disassembly operations, and storage in a batch mode. That, the evolution of the process is expressed using the graph of the $\mu$-markings of the BDSPN model, which represents this mechanism. Then, we have presented the resolution of the associated stochastic process used to determine the probabilities of the various states. The use of these probabilities assesses the average performance of the system. Performance indices associated with the BDSPN model have been calculated. An analytic method has been presented and applied to our model to analytically evaluate the performance of our approach. After that, a new mathematical model has been put forward to minimize the sum of extra costs, disassembly operations and holding costs for multi types of products and under different batch quantities. The suggested multicriteria assessment PROMETHEE II has enabled decision-makers to compare the performance results of various disassembly scenarios to determine the optimal ones. A real case study of the manufacturing company CODIMATRA has been adopted to test the effectiveness of the proposed approach.

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