Designing Strategyproof Election Systems with Score Voting

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Mots-clés : *voting system, Strategyproofness.*

Participatory budgeting (BP) seeks to invite citizens to participate in the process of deciding how public money is spent. Porto Alegre, Brazil, in 1989, first employed this form of participatory democracy. Since then, it has been used by different cities worldwide, like Madrid, Seoul, Bogota, New York, and Paris. For instance, in 2016, Paris applied KP to allow citizens to vote on allocating a budget of 100 million Euros [2].

Goel *et al.* [6] introduced a variant of PB. The idea comes from the fact that applying PB is conceptually similar to solving the classical Knapsack problem, with the set of chosen budget items fitting a limited budget B while maximizing societal value [3]. In this scheme, each citizen votes for a subset of the objects such that the sum of the costs of the objects satisfies the budget constraint. They showed that this schema is strategyproof and welfare-maximizing when the outcome for the voter is given by the ℓ^1 distance from the outcome and its true preference and partially strategyproof under additive concave utilities. More general mechanisms may not be strategyproof due to an important impossibility result [7, 5, 8].

We formalize the different notions. An *election* is a tuple $(\mathcal{V}, \mathcal{O}, W, w, \{u_v\}_{v\in\mathcal{O}})$, where :

— a set of voters $\mathcal{V} = [n]$ (where $[i] = \{1, \ldots, i\}$, for $i \in \mathbb{N}$) and a finite set of m alternatives (objects) $\mathcal{O}$ are considered ;
— W is the available *budget* (a weight constraint that restricts valid solutions) ;
— Every object o has also a cost denoted $w(o)$ ($w : \mathcal{O} \to \mathbb{R}$) : each $o \in \mathcal{O}$ assigns the *cost* that needs to be paid if o is selected. For each $S \subseteq \mathcal{O}$, we write $w(S) = \sum_{o\in S} w(o)$ for the total cost of S. In our context, each object has the same cost ;
— Each voter $v \in \mathcal{V}$ has a private preference over all solutions given by a utility function [1] $u_v : \mathcal{P}(\mathcal{O}) \to \mathbb{R}$.

A subset of objects $S \subseteq \mathcal{O}$ is *feasible* if $w(S) \leq W$. Each voter v votes via a ballot b_v : a ballot is a feasible subset $e \in \mathcal{P}(\mathcal{O})$. A *declaration profile* $\mathbf{B} = (b_1, \ldots, b_n)$ consists of a ballot b_v for each voter v.

Our goal is to choose a feasible subset of candidates. An *aggregation rule* $\mathcal{A}lgo$, aggregates a set of individual ballots $\mathbf{B}$ into a collective decision and returns a *solution*, which is a *winning set* of objects $\mathcal{A}lgo(\mathbf{B})$.

Our study focuses on the design of the social choice function that incentives an individual voter to vote sincerely. We focus on a social welfare *utilitarian* function (it maximizes $\sum_{v\in\mathcal{V}} \sum_{o\in s} b_v(o)$).

We denote by b_v^* the ballot of the voter v if she sincerely answers the questions based on her utility. Given an election, an aggregation rule is *strategyproof* if : for all profile ballots $\mathbf{B}$, for all voters $v \in \mathcal{V}$, we have $u_v(\mathcal{A}lgo(b_v^*; \mathbf{B}_{-v})) \geq u_v(\mathcal{A}lgo(b_v; \mathbf{B}_{-v}))$.

Our work focuses on a particular voting system using *score aggregation rules.* Score Voting can be used to model election systems. The model is similar to Knapsack Voting [6], where every ballot that contains an object o gives one point to o, and the winner is the object with

1. The set of all subsets of S is denoted by $\mathcal{P}(S)$.

the most points. The idea of a score function is to generalize the votes, allowing voters to give points to any given object.

A *score aggregation rule* $\mathcal{A}lgo$ on m objects is defined by a $m \times m$ real matrix M and an associated social choice function $\mathcal{A}lgo$. Given an integer W, and a ballot profile $\mathbf{B} = (b_1, \ldots, b_n)$, with $b_1, \ldots, b_n$ seen as a column vector of m elements, $\mathcal{A}lgo$ taking $\mathbf{B}$ as input returns the winning set of W objects which maximizes the inner product $M \cdot \mathbf{e}$ where the i^{th} element of vector $\mathbf{e}$ represents the number of times where o_i is in $b_1, \ldots, b_n$. If such objects are not clearly defined, we use the strict order given by $\mathcal{A}lgo$ as a tie-breaker. The voters must vote for W objects, but they can vote multiple times for the same object.

Example : The city council would like to propose four projects to the residents, such as "renovating a library" (o_1), "creating a bike path" (o_2), "funding a soccer team" (o_3), or "funding a basketball team" (o_4). However, it can only fund two of the four and wants to avoid funding two sports-related projects. Thus, it can construct a score function with $M = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & 3 \end{pmatrix}$. The ballot distribution is the following : four voters want to fund only the sports projects, two want to fund only the non-sports projects, and two want to fund the "creating a bike path" and "funding a basketball team". The following vector is summarized by $V = \begin{pmatrix} 2 \\ 4 \\ 4 \\ 6 \end{pmatrix}$. Since $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \\ 4 \\ 10 \end{pmatrix}$, the winner set is {"creating a bike path", "funding a basketball team"} because the scores of these objects are the highest.

We prove that :

— the neutral[2] score aggregate rules are equivalent to knapsack voting on the same instance ;

— any score aggregate rule is strategyproof if and only if its score function satisfies the *Constrained Change Property (CCP)* as follows : if a voter v switches from an object α to an object β in a ballot, the social choice function can either :
  - — eject α of the output solution and have it replaced by another object ;
  - — get β to be chosen and eject a previously chosen object (and only one) ;
  - — does not change the output solution.

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2. Election systems that treat objects equally.