Multi-Period Employee Scheduling and Routing: Formal Languages-based formulations

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1 Introduction

Multi-Period Scheduling and Routing (MPSRP) refers to problems involving employee mobilisation to perform work-related activities at geographically dispersed customer locations over a period of several days. This problem consists of the integration of two NP-hard problems: (i) the personnel scheduling problem and (ii) the vehicle routing problem.

Each problem have been extensively studied in the literature. However, to the best of our knowledge, the integration of both of them when complicating work rules need to be included in a multi-period environment has not been previously addressed. Certain constraints in personnel scheduling such as work regulations lead to substructures of the problem including constraints on sequences of shifts. It has been shown that exploiting formal languages to model such substructures facilitates the formulation and provides models that are easier to solve [2]. Formal languages have already been successfully used to solve personnel scheduling problems where daily shifts are modeled with context-free grammars [3]. However, in practice, there are certain types of constraints (e.g., the total working length over the planning horizon), here called *global constraints*, that are difficult to integrate with this modeling paradigm.

To address the aforementioned issue, we propose a generic approach (based on formal languages) to model and to include work regulations in MPSRP. This approach allows to incorporate a great variety of work rules including *global constraints*. The personnel scheduling component of the problem is coupled with an extended vehicle routing problem (here called E-VRPTW) including customers' time windows, skill requirements and a set of complex scheduling-related constraints. To evaluate the performance of our approach we compare and discuss three modeling methods: (i) a classical MIP formulation, (ii) a grammar-based formulation with supplementary constraints and (iii) a new automaton-based formulation encapsulating all work rules.

2 Automaton-based formulation for personnel scheduling

Given a set of scheduling-related constraints $\mathcal{C}$ and a time horizon D, we introduce a mathematical formalism and an algorithmic approach to build a *minimal* (i.e with as few states as possible) Deterministic Finite Automaton (DFA) whose language corresponds to the set of shifts sequences of length D respecting all constraints in $\mathcal{C}$. The structure of the obtained DFA is a $D+1$-layered graph with no cycles. From this DFA we derive a MIP formulation, similar to the one presented in [2]. This formulation allows to respect all the constraints in $\mathcal{C}$. Consequently, this approach allows to aggregate employees according to their profile (e.g., availability, skills), so that we can reduce the size of the personnel scheduling model, as the number of variables is proportional to the number of employee profiles.

3 Preliminary computational results

We propose to solve the MPSRP which includes constraints of customers' time windows, skill requirements and a set of work regulation rules (including *global constraints*). Since the goal of this work is to compare the formulations related to the personnel scheduling problem, we solve the E-VRPTW using a classical MIP formulation.

We then formulate the scheduling constraints either with a classical MIP formulation, or with a grammar-based formulation with supplementary constraints corresponding the *global constraints*, or with the automaton-based formulation encapsulating all work rules. Figure 1 shows a schema of the three structures we want to compare.

We test and compare the three models using several instances representing a home health care problem. These instances are built based on a daily instance generation method described in the literature [1]. Preliminary results already show the effectiveness of the formal formulation based on DFA.

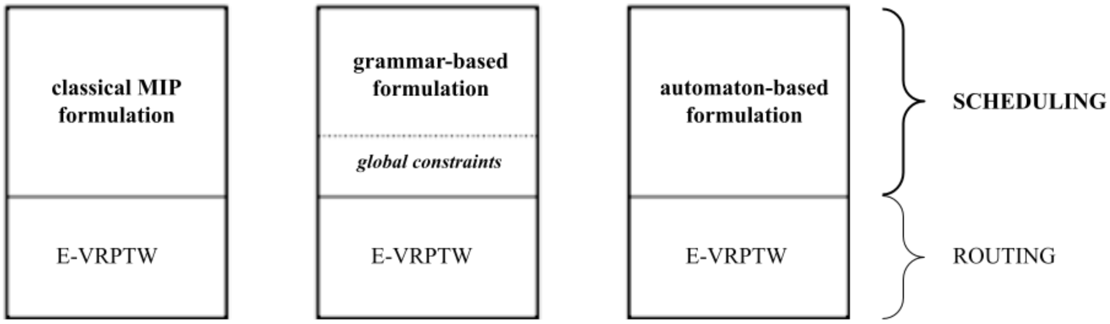

FIG. 1: Structures of the three formulations

4 Conclusion

We presented the mathematical formalism and algorithmic method allowing a formal-language based formulation of a complex set of constraints. We then modeled a complex MPSRP-type problem and solved various instances by comparing the performance of three formulations for the scheduling-related constraints: a classical MIP formulation, a context-free grammar formulation with supplementary *global constraints* and a complete formal formulation using DFAs.

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References

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