Inventory Control for Periodic Intermittent Demand

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1 Introduction

Intermittent demand is characterized by many periods with no demand at all, that are seemingly randomly interspersed with non-zero demand observations. The compound nature of intermittent demand complicates demand forecasting and efficient inventory management [1]. First, it is unknown when the next demand will occur. Second, there is uncertainty in the size of the demand when it does occur.

Specific approaches have been developed to tackle uncertainty in the demand. The analysis of intermittent demand times series is commonly decomposed to create separate estimates for the time between demand occurrences (the inter-demand interval) and the size of a demand occurrence, initiated by the work of Croston [2]. In the existing literature, these analyses implicitly assume that the time between demand occurrences is memoryless, i.e, the probability of observing a demand in a period is assumed to be independent of the time since the last demand observation. Data from practice, however, indicates that the times between demand events is often not memoryless, i.e., the probability of a demand occurrence does depend on the time since the last demand occurred. Consequently, the time since the last demand event is an important predictor for future demand.

2 Approach and contributions

This research is the outcome of a joint research project with a company in the chemical industry. This company observes intermittent demand for many of its items, and data suggests periodicity in the demand occurrences. We use the discrete compound renewal process to model such periodic intermittent demand. We consider a single stock-point under periodic review. Unsatisfied demand is backordered at a backorder penalty cost per time unit and inventory on-hand at the end of a period incurs a holding cost. We allow for positive lead times. We use a Markov decision process formulation to study the structure of optimal policies.

The main contributions of our work are characterizations of optimal order policies. We show that the optimal policy is a state-dependent base-stock policy, where the state is the time since the last demand observation. We prove this by induction on the value function. We also show that there exist state-dependent base-stock policies for which the optimal base-stock level is non-decreasing in the time since the last demand, regardless of whether the hazard rate of the time between demand arrivals is increasing, decreasing, or fluctuates. As such, any algorithm that searches for optimal (or good) base-stock levels can constrain the search space to non-decreasing base-stock levels only. Contrary to what may be expected, this result is not proven by showing sub-modularity of the value function. Instead we use a different approach to that exploits structure in compound renewal processes directly to characterize how optimal base-stock level can vary with the time since the last demand occurrence.

3 Results

We benchmark two fast heuristics against the optimal policy. The first heuristic is a myopic state-dependent base-stock policy and the second heuristic is a stationary base-stock policy. We identify settings where it is especially valuable to exploit the periodicity inherent in compound renewal processes. Overall, we find that both heuristic policies have optimality gaps in all scenarios. More specifically, we show that the optimality gap of both the myopic and the stationary policy increases when demand becomes more intermittent, when the coefficient of variation of the time between demands is small, and when the lead time is short. Furthermore, we observe that the myopic state-dependent policy consistently outperforms the stationary policy. These results show the significant value of correctly using information on demand periodicity and on the time since the last demand occurrence in the order decisions. Even when this information is only included in a myopic way, there are already large gains compared to when a stationary policy is applied.

We also evaluate and benchmark the performance of different approaches in real data. We use demand data from the chemical company mentioned above, as well as 4 data sets from the literature. These data sets represent diverse industries in which our model adds value. These industries are (i) specialty chemicals, (ii) air force spare parts, (iii) car parts, and (iv) retail. The retail and chemical industries are not well known for having items with intermittent demand. Shorter lead times and increased product differentiation/customization make intermittent demand increasingly common in these industries.

We compare the performance of the fast heuristics with the optimal policy. For the stationary policy, we consider two different approaches : the first separately considers the distribution of the time between demands and that of the demand sizes, and derives the stationary distribution from those. The second considers the distribution of the lead time demand directly. The distribution parameters of the time between demand occurrences, the hit sizes (upon demand occurrence), and the lead time demand are not known upfront and are estimated from the data using Maximum Likelihood Estimation. Additionally, we also consider the Syntetos-Boylan approximation method [3], which is widely considered in literature for intermittent demand forecasting.

Références

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