Local search algorithms for the robust vehicle routing problem with time windows and budget uncertainty

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Keywords : *robust optimisation, vehicle routing, metaheuristics.*

1 Introduction

The vehicle routing problem with time windows (VRPTW) is a relevant variant of the classic vehicle routing problem due to its numerous applications that emerge from real-life logistics necessities. Therefore, this problem has received substantial attention in the literature (see the survey [2]). The solution of VRPTW minimizes the overall operational costs in routes to serve visit points while respecting capacity constraints and intervals of service time (time windows).

In the deterministic version of VRPTW, all the input data are static and known in advance. However, in practice, the travel time may vary because of different sources of uncertainty. For example, accidents, traffic jams, or working roads usually add time delays. Hence, the models for the deterministic VRPTW may not be ideal in these cases. The literature on the VRPTW with uncertain travel times typically relies on two frameworks: stochastic programming and robust optimization. This work focuses on robust optimization, which often leads to a more tractable model than stochastic optimization, particularly for routing problems (e.g., [1, 3]).

2 The robust VRPTW model

In order to handle the uncertainty, our Robust VRPTW (RVRPTW) model considers that the travel times belong to a known finite set. The RVRPTW is defined on a directed graph $G = (V, A)$, where $V = V^* \cup \{o, d\}$, V^* is the set of clients to be visited, o is the origin depot, and d is the destination depot. Each vertex $i \in V$ is associated with a demand q_i and with a time window $[e_i, l_i]$. Set A contains arcs between pairs of vertices (i, j), and each arc has a travel cost c_{ij} and a travel time t_{ij}. In addition, let K represent the set of vehicles, where each vehicle $k \in K$ has a capacity C.

Next, we introduce two important sets. Let $X \subseteq \{0,1\}^{|A|\times|K|}$ be the set of feasible paths starting at o and ending at d, and $T^\Gamma \subseteq \mathbb{R}^{|A|}$ be the uncertainty set. Moreover, let $\bar{t}_{ij}$ and $\hat{t}_{ij}$ denote the nominal travel time and the deviation for arc $(i, j) \in A$. Further, let $\Gamma \in \mathbb{Z}$ denote the maximum number of components of t equal to the deviated value $\bar{t}_{ij} + \hat{t}_{ij}$ for any travel time vector $t \in T^\Gamma$. Formally, set T^Γ is defined as follows:

$$T^\Gamma = \left\{ t \in \mathbb{R}^{|A|} : t_{ij} = \bar{t}_{ij} + \delta_{ij}\hat{t}_{ij}, (i,j) \in A, \sum_{(i,j)\in A} \delta_{ij} \leq \Gamma, \delta_{ij} \in \{0,1\}, \forall (i,j) \in A \right\},$$

where δ_{ij} indicates whether arc (i, j) is deviated in the associated vector t.

Let x_{ij}^k be the binary variable that assumes 1 iff the vehicle $k \in K$ traverses the arc $(i, j) \in A$, and let $y_i(t)$ be a positive real variable indicating the arrival time at vertex i for a given uncertainty travel time parameter $t \in T^\Gamma$. Then, we can describe the RVRPTW as follows.

$$
\begin{aligned}
\min \; & \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ij}^k \\
\text{s.t. } & (x_{ij}^k = 1) \implies (y_j(t) \geq y_i(t) + t_{ij}), \quad (i,j) \in A, k \in K, \forall t \in T^\Gamma \\
& \sum_{k \in K} \sum_{(i,j) \in A} x_{ij}^k = 1, \quad \forall i \in V^* \\
& e_i \leq y_i(t) \leq l_i, \quad i \in V, \forall t \in T^\Gamma \\
& x \in X, y \in \mathbb{R}_+
\end{aligned}
$$

The formulation above typically leads to time-consuming algorithms by mathematical programming approaches (e.g., [1]). Hence, we will address it using Local Search (LS) algorithms.

3 Local search

A LS algorithm attempts to find improvements by locally modifying the current solution $x \in X$ so as to generate neighbor solutions in the search space. Moreover, an evaluation function measures the difference between solutions, guiding these modifications. Due to the numerous comparisons between solutions during the LS phase, efficient evaluations are crucial.

The Dynamic Programming (DP) approach presented in [1] checks the feasibility of robust time windows for a given $x \in X$ in polynomial time. However, in contrast to directly inserting this checker in the LS leading to a binary decision (feasible or infeasible solution), several methods consider penalizing infeasible solutions aiming to provide the distance among distinct solutions, some of them using the *time-warp* concept [4] to this end in the VRPTW.

In the context of RVRPTW, the adversary problem seeks to maximize penalization among all $t \in T^\Gamma$ leading to solving a maximization problem for the adversary. We conjecture that the problem of computing the maximum *time-warp* is $\mathcal{NP}$-Hard. Therefore, we propose the concept of *number of failures* in a route. Let $\phi_j(t, x)$ assumes 1 iff $y_i(t) + t_{ij} > l_j$ for a vertex $j \in x$, and a time $t \in T^\Gamma$, and 0 otherwise. Also, let $\phi(x)$ be the total of failures of a route x given by $\phi(x) = \max_{t \in T^\Gamma} \sum_{j \in x} \phi_j(t, x)$. As opposed to the concept of *time-warp*, the approach based on the number of failures leads to a polynomial time DP. We, therefore, intend to implement the proposed algorithm within an iterated local search framework and evaluate its performance against other methods on the benchmark instances.

References

[1] Agostinho Agra, Marielle Christiansen, Rosa Figueiredo, Lars Magnus Hvattum, Michael Poss, and Cristina Requejo. Layered formulation for the robust vehicle routing problem with time windows. In *International symposium on combinatorial optimization*, pages 249–260. Springer, 2012.

[2] Olli Bräysy and Michel Gendreau. Vehicle routing problem with time windows, part i: Route construction and local search algorithms. *Transportation science*, 39(1):104–118, 2005.

[3] Artur Alves Pessoa, Michael Poss, Ruslan Sadykov, and François Vanderbeck. Branch-cut-and-price for the robust capacitated vehicle routing problem with knapsack uncertainty. *Operations Research*, 69(3):739–754, 2021.

[4] Thibaut Vidal, Teodor Gabriel Crainic, Michel Gendreau, and Christian Prins. A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. *Computers & operations research*, 40(1):475–489, 2013.