Towards a Robust Multiobjective Master Surgical Schedule under Multiple Uncertainty

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1 Introduction

Patient scheduling within the operating room (OR) is subject to several forms of uncertainties related to the state of health of the patients and also to the availability of resources. Surgery duration and postoperative length-of-stay (LOS) in downstream recovery units, namely the intensive care unit (ICU) and the post-surgery units are two of the most significant patient-related sources of disturbance to the schedule. Elective patients undergo the surgery, then go to the post-surgery units. However, some patients must stay for one night or more in the ICU beds before being transferred to the post-surgery unit. Several existing works focus only on the upstream scheduling that concerns only the OR planning, which yields infeasible and sub-optimal schedules. On the other hand, the limited literature that handles the postoperative resources assumes that surgery duration and patients' LOS follow a well-known distribution. Unfortunately, it is challenging to use distributions to deduce surgery duration and LOS as is the case in stochastic approaches for medium and short-term planning. Consequently, we propose a new methodology for tactical planning using robust optimization. As stated in [1], some research papers assume that the OR manager has a unique objective to satisfy. In reality, the OR can be managed by a group of decision-makers that strive to satisfy conflicting objectives. This work focuses on building a robust master surgical schedule (MSS) for tactical planning. To do this, we assign elective patients to available ORs under the block scheduling strategy. Our approach considers multiple conflicting objectives: patient priority, assignment cost, and workload balancing. We consider several OR restrictions: OR resources, surgeons' and downstream resources availability while incorporating uncertainty in surgery duration and patients' LOS in the ICU.

2 Modeling surgery duration and LOS uncertainty

We develop a multiobjective programming model where capacity constraints are defined as follows:

$$\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{I}_s} d_i\chi_{isrj} \leq O^{\max} \qquad \forall r\in\mathcal{R} \qquad \forall j\in\mathcal{J} \tag{1}$$

$$\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{I}_s}\sum_{r\in\mathcal{R}}\sum_{\substack{j'\in\mathcal{J}\\ j'>j-l_{is}^{ICU}}}^{j} r_{is}\chi_{isrj'} \leq \nu_j \qquad \forall j\in\mathcal{J} \tag{2}$$

Where χ_{isrj} is a decision variable set to 1 if surgery $i\in\mathcal{I}_s$ (set of patients) of specialty $s\in\mathcal{S}$ (set of surgical specialty) is assigned to day $j\in\mathcal{J}$ (set of days) in room $r\in\mathcal{R}$ (set of ORs) and 0 otherwise. d_i represents the surgery duration. $O^{\max}$ is the capacity of the OR session. l_{is}^{ICU} represents the total LOS in the ICU for patient $i\in\mathcal{I}_s$. r_{is} is 1 if patient $i\in\mathcal{I}_s$ requires an ICU bed and 0 otherwise. Constraint (1) is the capacity constraint; it ensures that the capacity of each block time is respected. Constraint (2) guarantees daily beds' availability in the ICU. To address the uncertainty, we use the budgeted uncertainty set introduced by [2], which ensures the robustness of solutions without the necessity of generating scenarios. We assume that surgery duration is uncertain and modeled as an independent random variable that falls within the range $d_i \in [\overline{d}_i, \overline{d}_i + \widehat{d}_i]$ where $\overline{d}_i$ represents the nominal value and $\widehat{d}_i$ is the deviation ($\widehat{d}_i \geq 0$). The maximum worst-case value of surgery duration that we aim to protect is $\overline{d}_i + \widehat{d}_i$. The random variable denoted by λ is written as follows: $\lambda_i = \frac{d_i - \overline{d}}{\widehat{d}_i}$, and it takes value in $[0, 1]$. Similarly, the LOS are uncertain values modeled as follows: $l_{is}^{ICU} \in [\overline{l_{is}^{ICU}}, \overline{l_{is}^{ICU}} + \widehat{l_{is}^{ICU}}]$. The normalized deviation from the nominal ICU is as follows: $\eta_{is} = \frac{l_{is}^{ICU} - \overline{l_{is}^{ICU}}}{\widehat{l_{is}^{ICU}}} \quad \forall s \in \mathcal{S}\ \forall i \in \mathcal{I}_s$. We model surgery duration and LOS uncertainty using the polyhedral uncertainty set defined by [2]. Γ_d and Γ_l represent the budget of robustness (BOR). At most Γ_d (resp. Γ_l) variables of surgery duration (resp. LOS) assume their maximum value and all the others assume the central value of the uncertainty interval.

$$\Xi_{rj}^{d} = \left\{ d_i \in \mathbb{R}^n | d_i = \overline{d}_i + \widehat{d}_i \lambda_i, \sum_{i \in \mathcal{I}} \lambda_i \leq \Gamma_d, 0 \leq \lambda_i \leq 1 \right\} \tag{3}$$

$$\Xi_{j}^{ICU} = \left\{ l_{is}^{ICU} \in \mathbb{R}^n | l_{is}^{ICU} = \overline{l_{is}^{ICU}} + \widehat{l_{is}^{ICU}} \eta_{is}, \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}_s} \eta_{is} \leq \Gamma_l, 0 \leq \eta_{is} \leq 1 \right\} \tag{4}$$

It is worth mentioning that it is challenging to address the worst-case maximization in constraint (2), since the uncertainty is in the set of indexes of the constraint contrary to constraint (1). Since there is no existing method to address problems of this structure, we develop a novel modeling technique to deal with the problem using the polyhedral uncertainty set (4).

3 Experimental experience

BOR (d)	$\Gamma_d = 0$	$\Gamma_d = 0$	$\Gamma_d = 0$	$\Gamma_d = 0$	$\Gamma_d = 1$	$\Gamma_d = 5$	$\Gamma_d = 10$	$\Gamma_d = 1$	$\Gamma_d = 5$	$\Gamma_d = 10$
BOR (ICU)	$\Gamma_l = 0$	$\Gamma_l = 1$	$\Gamma_l = 5$	$\Gamma_l = 10$	$\Gamma_l = 0$	$\Gamma_l = 0$	$\Gamma_l = 0$	$\Gamma_l = 1$	$\Gamma_l = 5$	$\Gamma_l = 10$
Objective	9603*	9530*	9188	8985	9519*	9348*	9348*	9446	8883	8788

TAB. 1: Results with different values of BOR for an instance with 150 patients and 12 OR

We have implemented our robust model using Julia/JuMP and IBM CPLEX 20.1. The data was taken from a French hospital. The approach can be implemented by medium-sized hospitals that adopt the block scheduling strategy. The computations were executed on an Intel Xeon Gold with 56 cores at 2.7 GHz and 256 GB of RAM, with different combinations of $\Gamma_d, \Gamma_l \in \{0, 1, 5, 10\}$. A maximum running time limit of one hour and a maximum gap of 0.05 were imposed. (*) defines an optimal solution. Table 1 gives an example of the extensive computational experiments made, the only objective considered in the Table 1 is patients priority. Our robust approach demonstrates the price of robustness. The patients needing ICU are limited. The objective function is proven optimal and not impacted by uncertainty when Γ_l is greater than 5 because all the patients require their maximum LOS. Additional results will be presented at the conference.

References

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