Impact of large claims on the stability bound of a bivariate risk model

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1 Introduction

In ruin theory, stochastic processes are used to model the surplus of an insurance company and to evaluate its ruin probability, i.e., the probability that the total amount of claims exceeds its reserve. This characteristic is a much studied risk measure in the literature. In general, this measure is very difficult or even impossible to evaluate explicitly. Thus, different approximation methods have been proposed to estimate this characteristic (see [2]). However, its assessment is not evident in several cases since it cannot be found explicitly in closed forms. Besides, it is not easy to determine the parameters governing these models since they are often unknown or partially known. For all these reasons, the stability analysis becomes crucial for studying such models. Hence, it is necessary to obtain explicit stability bounds. The strong stability method, which was developed by Aïssani and Kartashov (1983) [1], makes it possible to clarify the conditions for which the ruin probability of the complex risk model (real model) can be approximated by the corresponding ruin probability in the simple risk model (ideal model). The application of the strong stability method in the risk theory has been discussed extensively in Kalashnikov (2000) [4], in this article the author determines the stability bounds in the univariate classical risk models using the strong stability[1], where he used the analysis of the stability of limit distributions of general Markov chains.

2 Strong stability of the two-dimensional classical risk model

2.1 The two-dimensional classical risk model

Consider an insurance company with two line of business. The evolution in time of the capital of this company is often modeled by the process of reserve $\{X(t), t \geq 0\}$ described by

$$X(t) = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} t - \sum_{j=1}^{N(t)} \begin{pmatrix} Z_j^1 \\ Z_j^2 \end{pmatrix} \qquad t \geq 0 \tag{1}$$

Where the initial reserve $u = (u_1, u_2)$ and the premium rate $c_i, i = 1, 2$ are real positive constants and $N(t)$ be the number of claims between time 0 and t, which follows a Poisson process with parameter λ. For fixed $i = 1$ or 2, $\{Z_j^i, j = 1, 2, \ldots\}$ are i.i.d claim size random variables with common distribution F_i. For simplicity, we assume that $\{Z_j^1, j = 1, 2, \ldots\}$ and $\{Z_j^2, j = 1, 2, \ldots\}$ are independent and furthermore, both of them are also independent of $\{N(t), t \geq 0\}$.

we consider the time of ruin $T_{sum} = \inf\{t/ \ \ X_1(t) + X_2(t) < 0\}$. T_{sum} means that the total of $X_1(t)$ and $X_2(t)$ will be negative for one or more times in the future. The corresponding ruin probability is denoted by

$$\Psi_{sum}(u) = \mathbb{P}\left(\ T_{sum} < \infty/(X_1(0), X_2(0)) = (u_1, u_2)\ \right) \tag{2}$$

2.2 Stability bound of the bivariate classical risk model

Moreover, we consider another two-dimensional classical risk model $X'(t)$ defined in the same manner as for $X(t)$, corresponding to the ideal risk model :

$$X'(t) = \begin{pmatrix} X'_1(t) \\ X'_2(t) \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} c'_1 \\ c'_2 \end{pmatrix} t - \sum_{j=1}^{N'(t)} \begin{pmatrix} Z'^1_j \\ Z'^2_j \end{pmatrix}, \quad t \geq 0. \tag{3}$$

We also define in the same way its corresponding ruin probability $\Psi'_{sum}(u)$.

The stability bound of the ruin probability of the bivariate classical risk model is summarised in the following theorem :

Theoreme 1 *(see [3]) Let $\Psi_{sum}(u)$ and $\Psi'_{sum}(u)$ the ruin probabilities associates to the the reserve processes $X(t)$ and $X'(t)$ respectively. Then, under assumption $\mu(a, a') < (1-\rho)^2$ we have*

$$\|\Psi_{sum}(u) - \Psi'_{sum}(u)\|_v \leq \frac{\mu(a, a')}{(1-\rho)\ \left(\ (1-\rho)^2 - \mu(a, a')\ \right)} \tag{4}$$

where $\rho = \mathbb{E}\left(\ \exp\{\epsilon(Z_1^1 + Z_1^2 - (c_1 + c_2)\theta_1)\}\ \right)$ *and*
$\mu(a, a') = 2\ \mathbb{E}\ \ e^{\epsilon}(Z_1 + Z_2)\ \ \ln|\frac{\lambda(c'_1+c'_2)}{\lambda'(c_1+c_2)}| + \|F_1 * F_2 - F'_1 * F'_2\|_v$.

3 Contribution

We often deal in insurance with large claims that are described by heavy-tailed distributions (Pareto, Lognormal, Weibull, etc.) (see [5]). It is worth to notice the special importance of heavy-tailed distributions, which is increasing the last years because of appearance of huge claims ([5]).

In this work, we consider the problem of stability of the bivariate classical risk model, in particular at the stability bound of the classical risk model established by Benouaret and Aïssani (2010) [3] using the approache based on the strong stability method developed for general Markov chains [1]. We are interested in the stability bound defined in Theorem 1 by relation (4). The objective is to evaluate numerically the stability bound of the ruin probability considering a different heavy-tailed claim distributions, and to show the impact of large claims on the stability bound of a bivariate classical risk model.

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