New formulations for the inventory routing problem

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1 Introduction

The inventory routing problem – IRP arises from taking inventory and routing decisions together at an integrated level. It arises from a common business practice where suppliers manage customers' inventories by knowing their capacities, initial states and the demands for a defined planning horizon. In such a way, customers abdicate the effort of planning their inventories while suppliers can better arrange their deliveries, and both benefit from it. The IRP first appears in the literature in the seminal work of [3], which presents a computational decision support tool developed for planning the distribution of liquefied gases on a daily basis.

More recently, the problem has mostly been approached exactly by means of branch-and-cut algorithms. Regarding these algorithms, we briefly and not extensively point to [1], which introduces a replenishment policy and the literature's benchmark test set of instances for the IRP, [4] for solving the IRP with many additional features, and [2] for proposing a family of valid inequalities for the problem. At last, for a branch-cut-and-price for the IRP, we point to [5], which in addition to its formulation, also presents a procedure to separate the well-known rounded capacity inequalities for the problem.

2 Problem definition

We define the IRP over a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{A})$, where $\mathscr{V} = \{0\} \cup \mathscr{C}$ and $\mathscr{A} = \{(i,j) : i, j \in \mathscr{V}, i \neq j\}$. For every arc $(i,j) \in \mathscr{A}$, cost $c_{ij} = c_{ji} \geq 0$ is defined. We suppose that the triangle inequality holds. Node 0 corresponds to the supplier representation while set $\mathscr{C} = \{1, 2, \ldots, n\}$ represents customer nodes. The supplier must replenish customers over a finite time horizon $\mathscr{T} = \{1, 2, \ldots, T\}$ and at each of such time periods an amount p^t of goods becomes available. Inventory capacities C_i and initial inventories I_i^0, $i \in \mathscr{V}$, as well as customer demands d_i^t, $i \in \mathscr{C}$, are known in advance. For each unity of product stored in inventories at the end of each time period $t \in \mathscr{T}$, a unitary holding cost h_i applies. A fleet of H identical vehicles with capacity $Q > 0$ is at disposal for replenishing customers, which admit a single visit at each time period. Let $\mathscr{R}_t$ be the set of feasible routes in time period $t \in \mathscr{T}$. A route $r \in \mathscr{R}_t$ is *feasible* if the total delivered goods do not exceed Q. The cost c_r of route r is the sum of the costs of the arcs that it traverses. A feasible solution to the problem consists of a set of feasible routes in which the quantities delivered to customers $i \in \mathscr{C}$, through at most one visit per customer on each time period $t \in \mathscr{T}$, all together with their stocked goods from previous periods, suffice to avoid stock-outs. The objective of the IRP is to find a such feasible solution that minimises the sum of routing and storage costs.

Our first formulation models the special case in which all inventory holding costs are equal, thus $h_i = h, \forall i \in \mathscr{V}$. It encompasses the case with no holding costs. In this case, the sum of all holding costs $\sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{V}} I_i^t h_i$ can be written as $h \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{V}} I_i^t$. However, we show that the sum of all inventories at each time period can be determined by flow conservation, as the total production (inflow) and the total consumption (outflow) at each time period are known in advance. Thus, the sum of all storage costs can be calculated and is constant.

3 Solution approach

We propose a branch-cut-and-price – BCP algorithm with route-based formulations for the IRP. We show that only route variables suffice to model the problem in the case with equal costs. The validity of this formulation relies on an exponential family of non-robust inequalities, which can be efficiently separated exactly, ensuring correctness. To separate exactly these novel inequalities, we rely on representing solutions, either fractional or integer, through a suited flow graph, with as many supplier and customer representation nodes as time periods for the planning horizon. At each time period, the graph models how goods can flow from the supplier to customers, through arcs representing deliveries, or from inventory, through arcs linking a previous time-period representation of a given vertex to the next one. In addition, to strengthen our formulation, we implement separation heuristics for these novel non-robust inequalities, as well as for other known valid and robust inequalities for the problem, such as rounded capacity cuts. We show how this simpler approach can be extended to the general case with different holding costs, nevertheless the increased modelling difficulty. Computational results for the formulation with equal holding costs are expected for the conference.

References

[1] Claudia Archetti, Luca Bertazzi, Gilbert Laporte, and Maria Grazia Speranza. A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41:382–391, 8 2007.

[2] Pasquale Avella, Maurizio Boccia, and Laurence A. Wolsey. Single-period cutting planes for inventory routing problems. *Transportation Science*, 52:497–508, 6 2018.

[3] Walter J. Bell, Louis M. Dalberto, Marshall L. Fisher, Arnold J. Greenfield, R. Jaikumar, Pradeep Kedia, Robert G. Mack, and Paul J. Prutzman. Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13:4–23, 12 1983.

[4] Leandro C. Coelho and Gilbert Laporte. The exact solution of several classes of inventory-routing problems. *Computers & Operations Research*, 40:558–565, 2 2013.

[5] Guy Desaulniers, Jørgen G. Rakke, and Leandro C. Coelho. A branch-price-and-cut algorithm for the inventory-routing problem. *Transportation Science*, 50:1060–1076, 8 2016.