Agrégation de variables et symétries en PLNE

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Mots-clés : *Programmation Linéaire en Nombres Entiers, Symétries, Agrégation, Unit Commitment Problem*

1 Breaking symmetries by aggregating variables

Symmetries arising in integer linear programs can impair the solution process, in particular when symmetric solutions lead to an excessively large Branch and Bound (B&B) search tree. Various techniques, so called *symmetry-breaking techniques*, are available to handle symmetries in integer linear programs of the form $(ILP) \quad \min\{cx \mid x \in \mathcal{X}\}$, with $\mathcal{X} \subseteq \mathcal{P}(m,n)$ where $\mathcal{P}(m,n)$ is the set of $m \times n$ binary matrices. A symmetry is defined as a permutation π of the indices $\{(i,j) \mid 1 \leq i \leq m,\ 1 \leq j \leq n\}$ such that for any solution $x \in \mathcal{X}$, $\pi(x)$ is also solution with the same cost, *i.e.*, $\pi(x) \in \mathcal{X}$ and $c(x) = c(\pi(x))$. The *symmetry group* $\mathcal{G}$ of (ILP) is the set of all such permutations. It partitions the solution set $\mathcal{X}$ into *orbits*, *i.e.*, two matrices are in the same orbit if there exists a permutation in $\mathcal{G}$ sending one to the other. A *subproblem* is problem (ILP) restricted to a subset of $\mathcal{X}$. In [3], symmetries arising in solution subsets of (ILP) are called *sub-symmetries*. Such sub-symmetries may not exist in $\mathcal{G}$.

In this presentation, we consider symmetry-breaking techniques based on reformulations where integer variables are summed up along orbits. This type of reformulation aggregates variables, thus reducing the size of the resulting ILP [5]. Both symmetries and sub-symmetries are broken by this aggregated reformulation. In general, such a reformulation is a relaxation of the problem considered [4].

However in some particular cases, aggregated solutions can be disaggregated into integer feasible solutions, therefore leading to an exact reformulation. In this case, the *(ILP)* is said to have the *exact disggregation* property. This is for example the case when the integer decomposition property [1] holds. We will show that we can extend this property to more general cases. In some other cases, the (ILP) admits a feasible disaggregated solution for any optimal aggregated solution, but without optimality guarantee for the disaggregated solution. In this case, the *(ILP)* is said to have the *heuristic disaggregation* property. We will show that these properties will also be useful for symmetry-breaking aggregation in the context of a Dantzig-Wolfe reformulation [6].

When none of these disaggregation properties hold, the relaxation obtained by aggregation can still be useful. We will give some insights about its strength, and how to use it in a Branch&Bound framework.

2 Application to Unit Commitment Problems

In this presentation, the aggregation framework will be applied to different variants of the Unit Commitment Problem (UCP).

The first UCP variant considered features constraints on the minimum up and down times of each unit. This variant is called the Min-up/min-down Unit Commitment Problem (MUCP) as defined in [2], and is structurally close to French insular UCPs solved at EDF (in insular regions such as Guadeloupe, Martinique, Réunion, Guyane and Corse). When the MUCP is considered, the exact disaggregation property applies.

The second UCP variant considered is the *ramp-limited MUCP*, featuring minimum and down times, as well as power variation costs. This problem also appears in French insular UCPs. When the ramp-limited MUCP is considered, the heuristic disaggregation property applies.

The third UCP variant considered is the *ramp-constrained MUCP*, featuring minimum and down times, as well as constraints limiting power variations, referred to as *ramp constraints*. The French continental UCP for thermal units is structurally close to this variant. When the ramp-constrained MUCP is considered, no disaggregation properties apply.

Experimental results will be provided for both academic instances and real insular instances, showing the efficiency of the aggregated reformulation.

Références

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