# The lot sizing problem with profit maximization and discount

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## 1 Introduction

In an increasingly competitive environment, the essential challenge for companies is the continuous improvement of their performance. These depend mainly on the production of goods and services to meet customer requirements as soon as possible and to lower costs, while maximizing profits. In this paper we focused to integrate price in lot sizing problem with time windows and propose different discount prices to attract customers and maintain profit.

## 2 Problem description and modeling

### 2.1 Problem description

We consider a single-item dynamic lot-sizing problem of a manufacturer with pricing decisions on a planning horizon discretized in T periods without capacity. Following the assumptions in Lee and al. [2], we assume that there are N deterministic demands to satisfy, and each demand i has a time window [Ei; Li] within which the demand must be satisfied. However, contrary to Lee and al. [2], where the manufacturer can decide when to satisfy the demand, within its time window at no additional cost. we assume in this work that the customers prefer their demand to be satisfied in the first period of the time window, except if a discount is offered within the time window.in this case we have two price levels (price before discount and discount price) the customers systematically postpone their demand to the first period with a discount. The objective is to determine a production plan that shows the discount periods, and when the demand will be expressed for maximizing profit.

### 2.2 Problem modeling

*Parameters:*

- T:Number of periods , N: Number of demands,
- $d_i$: Quantity of demand i,
- $[E_i, L_i]$ :time window to satisfy $d_i$,
- $c_t$: Unit production cost in period t,
- $h_t$: Unit holding cost in period t,
- $s_t$: Fixed production setup cost in period t,
- P: Price without discount,$P^r$ : Price with discount,

*Variables:*

- $X_t$: Quantity to be produced in period t,
- $S_t$:: Inventory level in period t,
- $Y_t$:: Binary variable equal to 1 if and only if production takes place in period t and 0 otherwise,
- $z_t^i$: Binary variable which is equal to 1 if demand $d_i$ is satisfied in period t and 0 otherwise,
- $r_t$:: Binary variable which is equal 1 if t is a discount period and 0 otherwise,
- $v_t^i$: Binary variable which is equal 1 if $r_t = 1$ and $z_t^i = 1$ and 0 otherwise,

Our problem can be modeled using the following integer linear program:

$$MAX \sum_{t}^{T}\sum_{i=1}^{n}(pd_i z_t^i - (p-p^r)d_i v_i^t) - \sum_{t=1}^{T}(c_t X_t + h_t S_t + s_t Y_t) \quad (1)$$

$$X_t + S_{t-1} = \sum_{i=1}^{n} d_i z_t^i + S_t \qquad \forall t = 1 \cdots\cdots T \quad (2)$$

$$X_t \le \sum_{i=1}^{n} d_i Y_t \qquad \forall t = 1 \cdots\cdots T \quad (3)$$

$$\sum_{t=E_i}^{L_i} z_t^i = 1 \qquad \forall \mathrm{i} = 1 \cdots\cdots \mathrm{N} \quad (4)$$

$$\sum_{t=1}^{T} z_t^i = 1 \qquad \forall \mathrm{i} = 1 \cdots\cdots \mathrm{N} \quad (5)$$

$$z_t^i \ge r_t - \sum_{t'=E_i}^{t-1} r_{t'} \qquad \forall t \in [E_i, L_i] \;\; \forall \mathrm{i} = 1 \cdots\cdots \mathrm{N} \quad (6)$$

$$z_{E_i}^i \ge 1 - \sum_{t'=E_i+1}^{L_i} r_{t'} \qquad \forall \mathrm{i} = 1 \cdots\cdots \mathrm{N} \quad (7)$$

$$v_t^i \ge r_t + z_t^i - 1 \qquad \forall t \in [E_i, L_i] \;\; \forall \mathrm{i} = 1 \cdots\cdots \mathrm{N} \quad (8)$$

$$Y_t, r_t, z_t^i, v_t^i \in \{0,1\}, (S_t, X_t) \ge 0 \qquad \forall t = 1 \cdots\cdots T \;\; \forall \mathrm{i} = 1 \cdots\cdots \mathrm{N} \quad (9)$$

### 2.3 Numerical results

The instances used in our tests are taken from [1]. Numerical tests are performed on instances generated by varying parameters. Some results illustrated in (table 1) in case of T=24 N =14.

| **Price** | **Classical Uncapacitated Lot Sizing** | | | | **Uncapacitated Lot Sizing with discount price** | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Profit | Ecart (revenus-costs) | Times(s) | Gap | Discount | Profit | Ecart | Times(s) | Gap |
| **30** | 1478 | 8.20% | 0.053 | 0 | 29.00% | 1650.6 | 9.20% | 19.541 | 0 |
| **40** | 7998 | 44.20% | 0.055 | 0 | 22.00% | 8110.4 | 45.10% | 7.704 | 0 |
| **50** | 14518 | 80.30% | 0.102 | 0 | 17.50% | 14660.5 | 81.70% | 7.476 | 0 |
| **80** | 34078 | 188.50% | 0.075 | 0 | 12.50% | 34078.0 | 188.50% | 6.802 | 0 |
| **100** | 47118 | 260.60% | 0.084 | 0 | 8.50% | 47411 | 266.50% | 10.183 | 0 |

TAB. 1 – comparison between the results obtained in the case of classic lot sizing and lot sizing with discount for different selling prices.

## 3 Conclusions et perspectives

The results obtained from our tests are interesting, this pushed us to do other tests by varying the parameters, and to solve the problems of very large sizes by proposing different discounts price.

## References

[1] N. Brahimi and S. Dauzère-Pérès, «A lagrangian heuristic for capacitated single item lot sizing problem,» 4OR, vol. 13, n° 2, p. 173–198, 2015.

[2] C. Y. Lee, S. Cetinkaya et A. P. M. Wagelmans, «A dynamic lot-sizing model with demand time windows,» Management Science, vol. 47, n° 110, pp. 1384,1395, 2001.