Adaptive Cutting Plane Selection with Analytic Centers

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1 Introduction

Cutting planes are an essential component of modern Mixed-Integer Programming solvers, they tighten the convex relaxations around integer-feasible points and improve the dual bound at the root node and throughout the tree. A well-designed cutting plane separation procedure often helps to reduce the branch-and-bound tree size while accelerating the overall solving process. The selection of cuts is a central trade-off: too little cutting leads to large branch-and-bound trees; too many cuts to a small node throughput and numerical instability. Among the typical criteria used for cut selection, a key measure represents the "strength" of the cut, i.e. it is a surrogate for how much of the relaxation is removed or for the dual bound increase, with *efficacy* being the canonical measure implemented in most solvers.

2 New distance measures for cutting plane selection

We will present properties of the distance measures used for cut selection and define dominance-consistency, a soundness property that is desirable for all cut measures and detail cases in which it holds and does not. We study two cases that hinder the applicability of some distance measures, namely dual degeneracy and infeasible Euclidean projection of the relaxation solution onto the cut. Both of these cases occur very frequently during the separation of real instances, and the latter results in dominance consistency not holding for efficacy. Motivated by such cases, we propose new tractable distance-based measures that are more robust to different geometries of the problem. These newly-proposed measures use the analytic centers of the relaxation polytope or of its optimal face, as well as alternative optimal solutions of the relaxation.

3 Performance evaluation and prediction

We assess the impact of the distance measure on root-node and tree-wide solver performance, comparing our measures against those prevalent in the literature. Unlike previous beliefs, the choice of distance measure significantly impacts solver performance both in time and number of nodes, with different measures performing better on different groups of instances. Motivated by this observation, we design a multi-output regression predicting the relative performance of each measure using static features readily available before the separation process. Our results indicate that analytic center-based methods help to significantly reduce the number of branch-and-bound nodes needed to explore the search space and that our multiregression can further improve on any individual method by selecting the most promising measure for a new instance.