# Robust inventory management under joint demand and lead time uncertainty

Benoit Loger[1], Alexandre Dolgui[1], Fabien Lehuédé[1], Guillaume Massonnet[1], Stefan Minner[2,3], Burakhan Sel[2]

[1]IMT Atlantique, LS2N
4 rue Alfred Kastler - La Chantrerie - 44307 Nantes - France
{benoit.loger,alexandre.dolgui,fabien.lehuede,guillaume.massonnet}@imt-atlantique.fr
[2]TUM School of Management, Chair of Logistics and Supply Chain Management
Technical University of Munich
Arcisstraße 21 - 80333 Munich - Germany
{stefan.minner,burakhan.sel}@tum.de
[3]Munich Data Science Institute
Technical University of Munich
Walther-von-Dyck-Straße 10 - 85748 Garching bei München - Germany

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## 1 Introduction

Today, the growing importance given to the reactivity of production systems and service level places the efficiency of supply systems at the centre of companies' concerns. Among the various factors influencing the performances of supply systems, the impact of different sources of uncertainty has been the focal point of many research work for many years. Most of the existing works consider uncertain demand or delivery lead time. Although some works are interested in the integration of these two sources of uncertainty [1], these works are most often based on restrictive assumptions : 1) demand and lead time distributions are assumed to be known 2) discrete lead time are approximated by mean of continuous distributions 3) autocorrelation of demand and dependencies between these two sources of uncertainty are ignored. In many real-world problems, uncertain demand follow autocorrelation patterns, and in some cases, the lead times are influenced by the demand of previous periods. Indeed, a succession of high demand leads to a saturation of suppliers and thus increases delivery lead time. In practice, not taking these correlations into account in procurement policies can lead to sub-optimal decisions and loss of efficiency.

## 2 Robust optimization approach

In supply problems, it is not uncommon for the classical models aiming to optimise the average performance of systems to be replaced by approaches allowing performance guarantees to be obtained. Although the first works in this direction date back to the 1950s [6], the recent development of Robust Optimisation ([2] and [3]) have led to the development and application of new approaches to supply management. [4] applied the approach of [3] to a single-product, multi-period supply problem with uncertain demand in single- and multi-echelon supply systems. Since then, a lot of papers have extended their works and studied robust approaches to inventory problems. [5] have extended this work using different uncertainty sets. However, these latest contributions focus mainly on product demand uncertainties. The problem of delivery lead time uncertainty has been little studied in a robust context. The

work of [7] is the only one to propose a robust approach by considering uncertainties on the demand and the delivery lead time. Unlike the previous work, the robust model is solved using a column and constraint generation method, allowing us to obtain less conservative robust solutions. However, uncertainties in demand and delivery lead time are modelled by two separate uncertainty sets, ignoring possible dependencies between these two sources of uncertainty, and the demand is supposed to be i.i.d. over the different periods.

## 3 Contribution and initial results

Our work focuses on developing robust models and the definition of uncertainty sets that allow the integration of demand and delivery lead time uncertainties. We consider a periodic review inventory model for a single product over a finite planning horizon when demand backlogging is allowed. The obtained robust model defines the quantity to order at each period and is solved with a constraint and column generation algorithm. The objective function minimizes the sum of unit ordering costs, holding costs, and backlogging costs over the planning horizon. We study the impact of different correlation assumptions on the proposed robust solutions (demand autocorrelation, dependencies between demand and delivery lead time) and propose different approaches to incorporate these correlations into an uncertainty set that jointly controls demand and delivery lead time. The proposed solutions are evaluated and compared according to economic criteria such as worst and average costs as well as service level.

## Références


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