Relative Regret Single Machine Scheduling for Minimizing Maximum Lateness with Interval Data

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Keywords: Scheduling problems · Robust optimization · Relative Regret Criterion · Interval Data

1 Introduction

Uncertainty is involved in most real-life scheduling problems. One successful approach to handle uncertainty in problem data is *robust optimization* [1], which aims at producing decisions that have a reasonable objective value under any possible realization of parameters. The first critical element in the application of this approach is the choice of the structure of *the uncertainty set*, i.e., *the scenario set*. For scheduling problems, the most popular representation of the uncertainty is the *interval uncertainty*, where the value of each uncertain parameter is known to fall within a closed interval. The second critical element of the robustness approach is the choice of the appropriate robustness criterion. Three robustness criteria have been proposed in the literature : 1) *the minmax criterion* which aims at obtaining the best possible performance in the worst case scenario. 2) *the minmax regret criterion* which aims at minimizing the worst-case deviation from optimality over all possible scenarios. 3) *the minmax relative regret criterion* which aims at minimizing the worst-case percentage deviation from optimality over all possible scenarios. In this paper, we consider the minmax relative regret version of the problem of scheduling n jobs on a single machine to minimize the maximum lateness. Interval uncertainty about processing times and delivery times is taken into account. A polynomial algorithm for this problem is constructed.

2 Related works

The deterministic version of the considered scheduling problem is denoted in the three-field notation as $1 \mid\mid L_{max}$ and can be solved in $O(n)$ time by applying the Jackson's rule, i.e., sequencing the jobs in order of non-increasing delivery times. Most of the literature in robust combinatorial optimization is devoted to minmax regret criterion. For instance, Kasperski [2] developed a polynomial time algorithm for the minmax regret version of $1 \mid prec \mid L_{max}$ where interval uncertainty is related to processing times and due dates. However, minmax relative regret criterion has been little studied in the literature (see, e.g., [3]).

3 Problem definition and notation

We consider the problem of scheduling a set $\mathcal{J}$ of n non-preemptive jobs on a single machine. Each job is available at time 0 and is characterized by a *processing time* and a *delivery time*, which are not known in advance. However, an estimation interval for each value is known. Specifically, given a job $j \in \mathcal{J}$, let $[p_j^{\min}, p_j^{\max}]$ and $[q_j^{\min}, q_j^{\max}]$ be the uncertainty intervals for the processing time and the delivery time of j, respectively. A *scenario* $s = (p_1^s, ..., p_n^s, q_1^s, ..., q_n^s)$ is a possible realisation of all values of the instance, such that $p_j^s \in [p_j^{\min}, p_j^{\max}]$ and $q_j^s \in [q_j^{\min}, q_j^{\max}]$, for each $j \in \mathcal{J}$.

The set of all scenarios is denoted by $\mathcal{S}$. A solution is represented by a feasible *sequence* of jobs, $\pi = (\pi(1), ..., \pi(n))$ where $\pi(j)$ is the jth job in the sequence π. The set of all feasible sequences is denoted by Π. The maximal lateness in sequence $\pi \in \Pi$ under scenario $s \in \mathcal{S}$ is denoted as $L(\pi, s)$ and the value of an optimal schedule under a fixed scenario s, as $L^*(s)$. The job $c \in \mathcal{J}$ of maximum lateness in π under s is called *critical*. We denote by $B(\pi, j)$, $j \in J$, the set of all the jobs processed before job j in π, including job j.

In this paper, the minmax relative regret is illustrated by a game between two agents Alice and Bob. Alice selects a sequence π of jobs. The Bob's problem is defined for every feasible sequence π chosen by Alice and consists in selecting a scenario s such that the relative regret (of Alice) is maximized, i.e.,

$$Bob(\pi) = \max_{s \in S} \frac{L(s, \pi)}{L^*(s)} \tag{1}$$

The scenario that maximises the regret in (1) is called the *the worst-case scenario* for π. The Alice's problem consists in finding a sequence π which minimizes the maximum relative regret, i.e.,

$$\min_{\pi \in \Pi} Bob(\pi) \tag{2}$$

4 Our results

Lemma 1. *Let π be a sequence of jobs. There exists (1) a worst case scenario s for π, (2) a critical job c_π in π under s, and (3) a critical job c_σ in σ under s, where σ is the optimal sequence for s, such that:*

- *i for each job $j \notin B(\pi, c_\pi)$, it holds that $p_j^s = p_j^{\min}$,*
- *ii for each job $j \in \mathcal{J} \setminus \{c_\pi\}$, it holds that $q_j^s = q_j^{\min}$,*
- *iii for each job $j \in B(\pi, c_\pi) \cap B(\sigma, c_\sigma)$, it holds that $p_j^s = p_j^{\min}$, and*
- *iv c_σ is the first critical job in σ under s, i.e., c_σ processed before all the other critical jobs in σ.*

Theorem 1. *Bob calculates the maximum relative regret $Bob(\pi)$ of a given sequence π by guessing c_π the critical job in π, c_σ the first critical job in σ, where σ is the optimal sequence for s, and $k \in [\![1, n]\!]$ the position of job c_π in σ and solving for each guess (c_π, c_σ, k) a linear program with $n + 1$ variables and $4n + 5$ constraints.*

Theorem 2. *The optimal sequence of Alice π^* can be calculated in $O(n^5.T_{Bob}(n))$ time where $T_{Bob}(n)$ is the complexity of the Bob's problem.*

5 Conclusion

In this paper, we have developed a polynomial algorithm for the relative regret version of the $1 \,||\, L_{max}$ problem where the interval uncertainty is related to processing times and delivery times. This paper extends our work in [4].

References

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