Dual Ascent in Column Generation for Multi-Commodity Network Flow problems

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1 Introduction

Multi-Commodity network Flow (MCF) is a well-known class of Optimization problems. It refers to the problem of routing a set of commodities through a capacitated network, from their source to their target node, so that the capacity limits on the network arcs are respected and the overall commodity transportation cost is minimized. In the *Unsplittable* version (UMCF) all commodities must be routed but a commodity can only be routed through a single path from its source to its target. Also the *max-acceptance* version of the UMCF problem, in which a cost is applied for non-routed commodities, can be expressed as a UMCF problem. The UMCF problem is of practical relevance in various contexts, such as transportation, telecommunication and computer applications. Even though the UMCF can be formulated and solved as a compact integer linear program, it is known that even computing the linear relaxation of this compact formulation may be intractable for very large networks such as realistic telecommunications networks. The linear relaxation is usually solved much more efficiently using a path-flow formulation and a delayed column generation (CG) method [3]. However, instances of large size (large number of arcs, several commodities) may nevertheless require a considerable computational time, which could be reduced by more refined approaches.

A *dual ascent* algorithm is a heuristic procedure to solve the Lagrangian dual of a linear problem by exploiting its specific structure. It typically involves simple but problem-dependent computations to adjust the Lagrangian multipliers, and needs less iterations than a sub-gradient algorithm. Such an approach has been studied for different classes of problems, and in [1] it has been applied to the MCF problems. An extension of the dual ascent approach, based on a suitable reformulation of the problem, has been presented for the Set partitioning problem in [2] and it has been generalized to a wider class of problems in [4]. Results in those classes of problems are promising, since nearly optimal solutions are obtained in a shorter time than with a linear solver-based method.

2 Dual Ascent Algorithm for MCF

We extend the reformulation-based dual ascent approach presented in [2, 4] to the MCF problem. More specifically, we consider the path-flow formulation of the linear relaxation of the UMCF problem and we solve it with a column generation. We apply a suitable reformulation to each restricted master problem (RMP) in such a way as to obtain a Lagrangian relaxation that can be decomposed into smaller and easier sub-problems. The proposed reformulation is obtained by adding to each path variable a set of new variables, which correspond to each arc of the path, and expressing each path variable as a weighted average of the newly introduced variables. Moreover, constraints valid for the original RMP formulation are added in the reformulation and hence in the sub-problems, thus improving the Lagrangian bound. A way to retrieve a feasible solution for the dual of the RMP, exploiting the reformulation, is then presented: the bound provided by this dual solution can be stronger than the one of the Lagrangian relaxation obtained without the reformulation. Since it is based on the solution of trivial sub-problems, this dual solution is obtained effectively and this method, combined with a sub-gradient algorithm to improve the Lagrangian multipliers, allows to have a high quality dual solution of the RMP rapidly. The dual solutions obtained at each column generation iteration can be directly used to compute the pricing sub-problem: indeed dual feasibility assures that, if a column is found when performing the pricing step, adding it will improve the RMP domain. We notice that in this way we solve the column generation heuristically, but with good solutions, and in a short time; thus, a good dual bound for the linear relaxation of the problem is solved more efficiently than with a classical column generation method based on a linear solver. In order to have an exact solution method, we simply need to call a linear solver at the last RMP of the proposed column generation, starting from the dual variables provided. Also, a heuristic method to obtain a primal feasible solution is added, to measure the gap at each iteration.

Moreover, we propose a specific dual ascent step, in order to further reduce the number of sub-gradient steps by modifying the Lagrangian multipliers for each RMP. We show that this can be done by suitably increasing the value of specific Lagrangian multipliers associated with dualized capacity constraints. The dual ascent step can be combined with the reformulation-based solution, thus adding up the improvements in the number of iterations and computing time of the two proposed methods.

References

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