An exact method for a problem of time slot pricing

Olivier Bilenne[1], Frédéric Meunier[1]

CERMICS, École des Ponts ParisTech, France

{olivier.bilenne,frederic.meunier}@enpc.fr

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1 Problem statement

We consider a market where a population of customers is willing to purchase a service offered at different times $y_1, y_2, \dots, y_n$ under limited capacities. Our task is to anticipate and to implement a pricing profile $p = (p_1, \dots, p_n)$ that will optimally spread the expected hourly demand for the service. The time slot pricing problem is looked upon from the perspective of the service provider, in a competitive environment where profit maximization is a prime and vital concern for the company. Each customer has a preferred time $x \in \mathbb{R}$ for receiving the service, and the population is fully characterized by an absolutely continuous measure μ, with $\mu(A)$ being the volume of customers planning service during time interval A. In our model every customer is expected to either choose a time slot j that minimizes their total incurred cost $p_j + d(x - y_j)$, where p_j is the price paid for the service and the strictly convex function d is an expression of the inconvenience of rescheduling from their preferred time; or to reject the service if no time slot looks satisfying. The service plan induced by a pricing profile $(p_1, \dots, p_n) \in P^n$ is modeled by a collection $\tau_1, \dots, \tau_n$ of measures on $\mathbb{R}$, where $\tau_j(A)$ denotes the volume of the customers willing to be served during interval A and eventually getting the service at time y_j.

Our goal is to implement a pricing profile in P^n which maximizes the revenue of the service provider, defined as $p_1\tau_1(\mathbb{R}) + \cdots + p_n\tau_n(\mathbb{R})$, subject to maximal demand μ and to capacity limitations $\nu_1, \dots, \nu_n$, and under the assumption that all users seek to minimize their costs. This gives rise to a bilevel optimization problem which can be stated as the following mathematical program:

$$\begin{array}{lll} \text{maximize} & \sum_{k=1}^{n} p_k \tau_k(\mathbb{R}) & \\ \text{subject to} & \sum_{k=1}^{n} \tau_k \leqslant \mu & \\ & \tau_k(\mathbb{R}) \leqslant \nu_k & \forall k \in [n] \\ & \tau_k(A) > 0 \implies k \in \sigma_p(A) & \forall A \text{ measurable} \\ & p_1, \dots, p_n \in P & \\ & \tau_1, \dots, \tau_n \text{ measures on } \mathbb{R}\,, & \end{array} \tag{Prob}$$

where $\sigma_p(A) = \{\sigma_p(s) \colon s \in A\}$ denotes the image of set A under σ_p, and the set-valued function

$$\sigma_p(s) = \{k \in [n] \colon d(s - t_k) + p_k \leqslant \min(0, d(s - t_{k'}) + p_{k'})\ \forall k' \in [n]\}$$

embodies the lower-level problem faced by any user with preferred time s, as $\sigma_p(s)$ reports the time slots (if any) available for selection by the user under price profile p.

2 Contribution

Our contributions are mainly algorithmic. The decision variables of (Prob) share characteristics with the solutions of semi-discrete optimal transport problems. With these analogies in mind, we develop an exact method of solution based on dynamic programming. Our approach rests on encoding each candidate price profile p for (Prob) as a chain graph of length n with nodes $\{\Phi_k \equiv (i_k, j_k, p_k, q_k)\}_{k=1}^n$, in which i_k and j_k denote the last active (i.e., chosen by at least one user for service) time slot up to slot k and the first active time slot after k, respectively, and p_k and q_k are the price values assigned to these two time slots. Under the computational assumption that oracles are given for the value of d at any point and for the value of μ on any interval, the proposed algorithm produces a revenue-maximizing price profile in a number of operations polynomial in the size of P and the number of available time slots. In the case when the price set P is finite, we prove the following theorem.

Theorem 1. *When P is finite, problem* (Prob) *can be solved in time $O(n^4|P|^3)$.*

Our result extends to the case when $P = \mathbb{R}$ and μ has bounded support, through discretization of the price set. The main message then is that we can efficiently find solutions within arbitrary optimality gaps. Under strongly convex d, pointwise convergence of these bounds towards the optimum typically occurs at rate $O(\delta)$ linear in the discretization step δ, slowing down to $O(\sqrt{\delta})$ in some irregular cases where ϵ-optimality of the revenue can still be guaranteed within polynomial time.

Theorem 2. *Assume that d is strongly convex and that μ has bounded support where its density is lower bounded above zero. We can then compute in time $O(\frac{1}{\delta^3}n^4)$ lower and upper bounds* LB_δ *and* UB_δ *on the optimal value of* (Prob), *such that* $\mathrm{UB}_\delta - \mathrm{LB}_\delta = O(\delta^{1/2})$.

Among straightforward extensions of the proposed approach, we find the inclusion of additional operating costs, and users with differenciated or random service times. Numerical experiments are under way for a problem of time slot pricing at electric vehicle charging stations.

References

[1] F. Santambrogio. *Optimal transport for applied mathematicians.* Birk auser Springer, Basel, 2015.

[2] C. Villani. *Topics in Optimal Transportation.* American Mathematical Society, 2003.