

# Ordinal Power Indices : Lessons from a Real-World Dataset

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## 1 Ranking basketball players

Recently, the notion of *ordinal power indices* has been put forward in the literature. In a nutshell, the problem is as follows : given a ranking over coalitions of agents (or objects), how can we derive a ranking over singleton ? Among the possible application scenarios, team sports are often cited, as we typically observe group performances and may wish to derive individual contributions. One prominent such example is basketball.

Basketball is an interesting setting, since many different *line-ups* (subset of 5 players) are used throughout a given game. Also, there are a lot of available data for this sport. For instance, NBA play-by-play data is available (see for instance [2] for an example of how such data can be exploited in an AI context). Finally, there are existing rankings of players available which can be used for comparison, either based on expert assessment or data-oriented techniques (*e.g.* the Value Over Replacement Player).

## 2 Challenges for ordinal power indices

It is however important to emphasize some key differences with the theoretical framework of ordinal power indices, as studied *e.g.* in [1, 3] :

1. The observations do not consist of direct comparisons of line-ups of the same team  $\mathcal{T}$ . Instead, each line-up  $T_i \subset \mathcal{T}$  is opposed to different line-ups of *other* teams.
2. The observations can be more or less *certain*, depending for instance on the time played by the considered line-up.
3. The order is *partial*. In particular, only coalitions of size 5 are compared (and not all possible coalitions of size 5 are actually occurring, as there are constraints on positions filled by players).

We adopt a two-phase approach :

1. We seek to obtain, for a target team  $\mathcal{T}$ , an ordering over line-ups of this team. We compare different techniques to do so. A first approach (*scoring method*) is to evaluate, for each line-up, the point difference per minute. This provides a transitive ranking over the line-ups of a team. But this has the obvious drawback that line-ups may have faced opposing line-ups of very different strengths. At the other extreme we may require line-ups to be compared *ceteris paribus* : in that spirit, we propose the *witness method* where two line-ups  $T_i$  and  $T_j$  are compared by strictly identical witnesses, a witness being a line-up  $O_i$  of an opposing team which has been opposed to both  $T_i$  and  $T_j$ . Each witness votes for the pairwise comparison with a weight depending on the (minimum) time it has played against  $T_i$  and  $T_j$ . This results in a non-necessarily transitive majority graph, which can be turned into a transitive graph (*e.g.* by using the Copeland method).

2. Given a partial ranking over line-ups, we apply ordinal power indices methods to retrieve a ranking on players of  $\mathcal{T}$ . We may rely on existing techniques, such as CP-majority or Lexcel (in which case the tournament retrieved in 1. would need to be made transitive). To compare players  $x$  and  $y$ , the CP-majority solution will consider 4-player subsets as voters, yielding a majority graph on players. Several methods (Copeland, Top Cycle, etc.) can then be used to restore transitivity. The Lexcel solution ranks players by comparing (lexicographically) the number of times they appear in the different positions in the line-up ranking, from best to worst. We note that techniques based on the notion of marginal contribution are not well suited in our setting, as only line-ups of size 5 are available.

### 3 Some first observations

This paper is a preliminary report investigating the challenges raised by the application of ordinal power indices in such a real-world context. We studied the dataset of the 2009-2010 season, consisting of 1300 games, 30 NBA teams, with a mean number of 17 (min = 12, max = 24) players per team. The dataset shows an average of 441.4 effectively played line-ups per team (which represents a ratio of 5.15% when compared to the theoretical number of line-ups which could be formed per team). On average, a player plays in about 129 line-ups.

**Sparsity of witnesses.** Due to the high incompleteness of the ranking, the requirement of strictly similar witnesses for the witness method of phase 1. implies that many line-ups cannot be compared by the approach. On our dataset, it appears that about half of the pairs of line-ups do not have any single witness (*i.e.* these two line-ups never played against the same line-up during the whole season). Worse than that, more than 20% of the line-ups of a given team cannot be compared against any other line-up. A similar remark applies to the CP-majority method used in phase 2 : the requirement to have the exact same 4-player sub-line-up to express a preference among players means that only 20% of the available information is used, on average.

**Correlation of rankings.** We evaluate how close are the obtained rankings by using both the Kendall-tau and the Spearman distances. The Lexcel solution stands out as it provides rankings quite different from reference rankings. The rankings based on the different variants of CP-majority are significantly closer to the reference rankings, and constitute a set of very similar rankings.

This case study reveals that there are still several theoretical research topics to explore to handle such settings. For example, how to deal with incomplete data? How to account for preference intensity, or reliability degrees, in the initial data? And on the other hand, could we exploit more specifically the fact that the inputs only contains coalitions of the same size?

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### Références

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